-CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2012 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 *EITHER* State or imply non-modular inequality $(3(x-1))^2 < (2x+1)^2$
or corresponding quadratic equation, or pair of linear equations $3(x-1) = \pm(2x+1)$ B1
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values $x = \frac{2}{5}$ and $x = 4$ A1
State answer $\frac{2}{5} < x < 4$ A1

OR Obtain critical value $x = \frac{2}{5}$ or $x = 4$ from a graphical method, or by inspection, or by solving a linear equation or inequality B1
Obtain critical values $x = \frac{2}{5}$ and $x = 4$ B2
State answer $\frac{2}{5} < x < 4$ B1 [4]
[Do not condone $\le$ for $<$.]

2 *EITHER* Use laws of indices correctly and solve for 5^x or for 5^{-x} or for 5^{x-1} M1
Obtain 5^x or for 5^{-x} or for 5^{x-1} in any correct form, e.g. $5^x = \frac{5}{1 - {}^1/_5}$ A1
Use correct method for solving $5^x = a$, or $5^{-x} = a$, or $5^{x-1} = a$, where $a > 0$ M1
Obtain answer $x = 1.14$ A1

OR Use an appropriate iterative formula, e.g. $x_{n+1} = \frac{\ln(5^{x-1}+5)}{\ln 5}$, correctly, at least once M1
Obtain answer 1.14 A1
Show sufficient iterations to at least 3 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145) A1
Show there is no other root A1 [4]
[For the solution $x = 1.14$ with no relevant working give B1, and a further B1 if 1.14 is shown to be the only solution.]

3 Attempt use of $\sin(A+B)$ and $\cos(A-B)$ formulate to obtain an equation in $\cos\theta$ and $\sin\theta$ M1
Obtain a correct equation in any form A1
Use trig. formula to obtain an equation in $\tan\theta$ (or $\cos\theta$, $\sin\theta$ or $\cot\theta$) M1
Obtain $\tan\theta = \frac{\sqrt{6}-1}{1-\sqrt{2}}$, or equivalent (or find cost θ, $\sin\theta$ or $\cot\theta$) A1
Obtain answer $\theta = 105.9°$, and no others in the given interval A1 [5]
[Ignore answers outside the given material]

4 **(i)** Obtain correct unsimplified terms in x and x^3 B1 + B1
Equate coefficients and solve for a M1
Obtain final answer $a = \frac{1}{\sqrt{2}}$, or exact equivalent A1 [4]

(ii) Use correct method and value of a to find the first two terms of the expansion $(1+ax)^{-2}$ M1
Obtain $1 - \sqrt{2}x$, or equivalent A1 √
Obtain term $\frac{3}{2}x^2$ A1 √ [3]
[Symbolic coefficients, e.g. $\binom{-2}{1}a$, are not sufficient for the first B marks]
[The f.t. is solely on the value of a.]

5 **(i)** Use correct quotient or chain rule — M1
Obtain the given answer correctly having shown sufficient working — A1 [2]

(ii) Use a valid method, e.g. multiply numerator and denominator by sec x + tan x, and a version of Pythagoras to justify the given identity — B1 [1]

(iii) Substitute, expand $(\sec x + \tan x)^2$ and use Pythagoras once — M1
Obtain given identity — A1 [2]

(iv) Obtain integral $2\tan x - x + 2\sec x$ — B1
Use correct limits correctly in an expression of the form $a\tan x + bx + c\sec x$, or equivalent, where $abc \neq 0$ — M1
Obtain the given answer correctly — A1 [3]

6 Separate variables correctly and attempt integration of one side — B1
Obtain term $\ln x$ — B1
State or imply $\frac{1}{1-y^2} \equiv \frac{A}{1-y} + \frac{B}{1+y}$ and use a relevant method to find A or B — M1
Obtain $A = \frac{1}{2}$, $B = \frac{1}{2}$
Integrate and obtain $-\frac{1}{2}\ln(1-y) + \frac{1}{2}\ln(1+y)$, or equivalent — A1 √
[If the integral is directly stated as $k_1 \ln\left(\frac{1+y}{1-y}\right)$ or $k_2 \ln\left(\frac{1-y}{1+y}\right)$ give M1, and then A2 for $k_1 = \frac{1}{2}$ or $k_2 = -\frac{1}{2}$]
Evaluate a constant, or use limits $x = 2$, $y = 0$ in a solution containing terms $a\ln x$, $b\ln(1-y)$ and $c\ln(1+y)$, where $abc \neq 0$ — M1
[This M mark is not available if the integral of $1/(1-y^2)$ is initially taken to be of the form $k\ln(1-y^2)$]
Obtain solution in any correct form, e.g. $\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right) = \ln x - \ln 2$ — A1
Rearrange and obtain $y = \frac{x^2-4}{x^2+4}$, or equivalent, free of logarithms — A1 [8]

7 **(i)** *EITHER*: State or imply $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$ as derivative of $\ln xy$, or equivalent — B1
State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3, or equivalent — B1
Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ — M1
Obtain the given answer — A1

OR: Obtain $xy = \exp(1 + y^3)$ and state or imply $y + x\frac{dy}{dx}$ as derivative of xy — B1
State or imply $3y^2\frac{dy}{dx}\exp(1 + y^3)$ as derivative of $(1 + y^3)$ — B1
Equate derivatives and solve for $\frac{dy}{dx}$ — M1
Obtain the given answer — A1 [4]
[The M1 is dependent on at least one of the B marks being earned]

(ii) Equate denominator to zero and solve for y — M1*
Obtain $y = 0.693$ only — A1
Substitute found value in the equation and solve for x — M1(dep*)
Obtain $x = 5.47$ only — A1 [4]

8	**(i)**	Use correct product or quotient rule and use chain rule at least once	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve an equation with at least two non-zero terms for real x	M1	
		Obtain answer $x = \frac{1}{\sqrt{2}}$, or exact equivalent	A1	[4]
	(ii)	State a suitable equation, e.g. $\alpha = \sqrt{(\ln(4 + 8\alpha^2))}$	B1	
		Rearrange to reach $e^{\alpha^2} = 4 + 8\alpha^2$	B1	
		Obtain $\frac{1}{2} = e^{-\frac{1}{2}\alpha^2}\sqrt{(1 + 2\alpha^2)}$, or work *vice versa*	B1	[3]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.86	A1	
		Show sufficient iterations to 4 d.p. to justify 1.86 to 2 d.p., or show there is a sign change in the interval (1.855, 1.865)	A1	[3]

9	**(i)**	*EITHER*	Substitute $x = 1 + \sqrt{2}$ i and attempt the expansions of the x^2 and x^4 terms	M1	
			Use $i^2 = -1$ correctly at least once	B1	
			Complete the verification	A1	
			State second root $1 - \sqrt{2}$ i	B1	
		OR 1	State second root $1 - \sqrt{2}$ i	B1	
			Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}$ i	M1	
			Obtain $x^2 - 2x + 3$, or equivalent	A1	
			Show that the division of p(x) by $x^2 - 2x + 3$ gives zero remainder and complete the verification	A1	
		OR 2	Substitute $x = 1 + \sqrt{2}$ i and use correct method to express x^2 and x^4 in polar form	M1	
			Obtain x^2 and x^4 in any correct polar form (allow decimals here)	B1	
			Complete an exact verification	A1	
			State second root $1 - \sqrt{2}$ i, or its polar equivalent (allow decimals here)	B1	[4]

	(ii)	Carry out a complete method for finding a quadratic factor with zeros $1 \pm \sqrt{2}$ i	M1*	
		Obtain $x^2 - 2x + 3$, or equivalent	A1	
		Attempt division of p(x) by $x^2 - 2x + 3$ reaching a partial quotient $x^2 + kx$, or equivalent	M1 (dep*)	
		Obtain quadratic factor $x^2 - 2x + 2$	A1	
		Find the zeros of the second quadratic factor, using $i^2 = -1$	M1 (dep*)	
		Obtain roots $-1 + i$ and $-1 - i$	A1	[6]

[The second M1 is earned if inspection reaches an unknown factor $x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + (6/3)$ and an equation in A and/or B]
[If part **(i)** is attempted by the *OR 1* method, then an attempt at part **(ii)** which uses or quotes relevant working or results obtained in part **(i)** should be marked using the scheme for part **(ii)**]

10	**(i)**	*EITHER*	Use scalar product of relevant vectors, or subtract point equations to form two equations in a,b,c, e.g. $a - 5b - 3c = 0$ and $a - b - 3c = 0$	M1*
			State two correct equations in a,b,c	A1
			Solve simultaneous equations and find one ratio, e.g. $a : c$, or $b = 0$	M1 (dep*)
			Obtain $a : b : c = 3 : 0 : 1$, or equivalent	A1
			Substitute a relevant point in $3x + z = d$ and evaluate d	M1 (dep*)
			Obtain equation $3x + z = 13$, or equivalent	A1
		OR 1	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	M2*
			Obtain 2 correct components of the product	A1
			Obtain correct product, e.g. $12\mathbf{i} + 4\mathbf{k}$	A1
			Substitute a relevant point in $12x + 4z = d$ and evaluate d	M1 (dep*)
			Obtain $3x + z = 13$, or equivalent	A1
		OR 2	Attempt to form 2–parameter equation for the plane with relevant vectors	M2*
			State a correct equation e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 5\mathbf{j} - 3\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 3\mathbf{k})$	A1
			State 3 equations in x, y, z, λ and μ	A1
			Eliminate λ and μ	M1 (dep*)
			Obtain equation $3x + z = 13$, or equivalent	A1 [6]
	(ii)	*EITHER*	Find $\overrightarrow{CP}$ for a point P on AB with a parameter t, e.g. $2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + t(-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	B1 √
			Either: Equate scalar product $\overrightarrow{CP}, \overrightarrow{AB}$ to zero and form an equation in t	
			Or 1: Equate derivative for CP^2 (or CP) to zero and form an equation in t	
			Or 2: Use Pythagoras in triangle CPA (or CPB) and form an equation in t	M1
			Solve and obtain correct value of t, e.g. $t = -2$	A1
			Carry out a complete method for finding the length of CP	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 1	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Using a relevant scalar product find the cosine of CAB (or CBA)	M1
			Obtain cost $CAB = -\frac{22}{\sqrt{11}.\sqrt{62}}$, or cos $CBA = \frac{33}{\sqrt{11}.\sqrt{117}}$, or equivalent	A1
			Use trig to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 2	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Using a relevant scalar product find the length of the projection AC (or BC) on AB	M1
			Obtain answer $2\sqrt{11}$ (or), $3\sqrt{11}$ or equivalent	A1
			Use Pythagoras to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 3	State $\overrightarrow{AC}$ (or $\overrightarrow{BC}$) and $\overrightarrow{AB}$ in component form	B1 √
			Calculate their vector product, e.g. $(-2\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}) \times (-\mathbf{i} + \mathbf{j} + 3\mathbf{k})$	M1
			Obtain correct product, e.g. $-2\mathbf{i} + 13\mathbf{j} - 5\mathbf{k}$	A1
			Divide modulus of the product by the modulus of $\overrightarrow{AB}$	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1
		OR 4	State two of $\overrightarrow{AB}$, $\overrightarrow{BC}$) and $\overrightarrow{AC}$ in component form	B1 √
			Use cosine formula in triangle ABC to find cos A or cos B	M1
			Obtain $\cos A = -\frac{44}{2\sqrt{11}.\sqrt{62}}$, or $\cos B = \frac{66}{2\sqrt{11}.\sqrt{117}}$	A1
			Use trig to find the length of the perpendicular	M1
			Obtain answer $3\sqrt{2}$ (4.24), or equivalent	A1 [5]
			[The f.t is on $\overrightarrow{AB}$]	