Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/33**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

 

The following abbreviations may be used in a mark scheme or used on the scripts:

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| SOI | Seen or implied |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

**Penalties**

| | |
|---|---|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting. |

| Question | Answer | Marks |
|---|---|---|
| 1 | Commence division and reach a partial quotient $x^2 + kx$ | **M1** |
| | Obtain quotient $x^2 - 2x + 5$ | **A1** |
| | Obtain remainder $-12x + 5$ | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 2 | Plot the four points and draw straight line | **B1** |
| | State or imply that $\ln y = \ln C + x \ln a$ | **B1** |
| | Carry out a completely correct method for finding $\ln C$ or $\ln a$ | **M1** |
| | Obtain answer $C$ = 3.7 | **A1** |
| | Obtain answer $a$ = 1.5 | **A1** |
| | | **5** |

| Question | Answer | Marks |
|---|---|---|
| 3(i) | Calculate value of a relevant expression or expressions at $x$ = 2 and $x$ = 3 | **M1** |
| | Complete the argument correctly with correct calculated values | **A1** |
| | | **2** |
| 3(ii) | Use an iterative formula correctly at least once | **M1** |
| | Show that ($B$) fails to converge | **A1** |
| | Using ($A$), obtain final answer 2.43 | **A1** |
| | Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435) | **A1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 4(i) | Use correct $\tan(A \pm B)$ formula and express the LHS in terms of tan $x$ | **M1** |
| | Using $\tan 45° = 1$ express LHS as a single fraction | **A1** |
| | Use Pythagoras or correct double angle formula | **M1** |
| | Obtain given answer | **A1** |
| | | **4** |
| 4(ii) | Show correct sketch for one branch | **B1** |
| | Both branches correct and nothing else seen in the interval | **B1** |
| | Show asymptote at $x$ = 45° | **B1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 5(i) | State or imply $y^3 + 3xy^2 \frac{dy}{dx}$ as derivative of $xy^3$ | **B1** |
| | State or imply $4y^3 \frac{dy}{dx}$ as derivative of $y^4$ | **B1** |
| | Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer | **A1** |
| | | **4** |
| 5(ii) | Equate numerator to zero | ***M1** |
| | Obtain $y = -2x$, or equivalent | **A1** |
| | Obtain an equation in $x$ or $y$ | **DM1** |
| | Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$ | **A1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 6 | Separate variables correctly and attempt integration of one side | **B1** |
| | Obtain term $\tan y$, or equivalent | **B1** |
| | Obtain term of the form $k\ln\cos x$, or equivalent | **M1** |
| | Obtain term $-4\ln\cos x$, or equivalent | **A1** |
| | Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a\tan y$ and $b\ln\cos x$ to evaluate a constant, or as limits | **M1** |
| | Obtain correct solution in any form, e.g. $\tan y = 4\ln\sec x + 1$ | **A1** |
| | Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a\tan y$ and $b\ln\cos x$, and use correct method to find $x$ | **M1** |
| | Obtain answer $x = 0.587$ | **A1** |
| | | **8** |

| Question | Answer | Marks |
|---|---|---|
| 7(a) | Square $x + \mathrm{i}y$ and equate real and imaginary parts to 8 and –15 | **M1** |
| | Obtain $x^2 - y^2 = 8$ and $2xy = -15$ | **A1** |
| | Eliminate one unknown and find a horizontal equation in the other | **M1** |
| | Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent | **A1** |
| | Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent | **A1** |
| | | **5** |
| 7(b) | Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position | **B1** |
| | Show a circle with radius 2 and centre not at the origin | **B1** |
| | Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis | **B1** |
| | Shade the correct region | **B1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 8(i) | Use a relevant method to determine a constant | **M1** |
| | Obtain one of the values $A = 2$, $B = 2$, $C = -1$ | **A1** |
| | Obtain a second value | **A1** |
| | Obtain the third value | **A1** |
| | | **4** |
| 8(ii) | Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on $A$, $B$ and $C$] | **B2 FT** |
| | Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$ | ***M1** |
| | Use at least one law of logarithms correctly | **DM1** |
| | Obtain the given answer after full and correct working | **A1** |
| | | **5** |

| Question | Answer | Marks |
|---|---|---|
| 9(i) | Use correct product or quotient rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain a 3 term quadratic equation in $x$ | **M1** |
| | Obtain answers $x = 2 \pm \sqrt{3}$ | **A1** |
| | | **4** |
| 9(ii) | Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$ | ***M1** |
| | Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent | **A1** |
| | Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent | **A1** |
| | Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts | **DM1** |
| | Obtain the given answer | **A1** |
| | | **5** |

| Question | Answer | Marks |
|---|---|---|
| 10(i) | Equate at least two pairs of components of general points on *l* and *m* and solve for $\lambda$ or for $\mu$ | M1 |
| | Obtain correct answer for $\lambda$ or $\mu$, e.g. $\lambda = 3$ or $\mu = -2$ ; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$ | A1 |
| | Verify that not all three pairs of equations are satisfied and that the lines fail to intersect | A1 |
| | | 3 |
| 10(ii) | Carry out correct process for evaluating scalar product of direction vectors for *l* and *m* | *M1 |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | DM1 |
| | Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians | A1 |
| | | 3 |
| 10(iii) | *EITHER:* Use scalar product to obtain a relevant equation in *a*, *b* and *c*, e.g. $-a + b + 4c = 0$ | B1 |
| | Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$ | M1 |
| | Obtain $a : b : c = 2 : -2 : 1$, or equivalent | A1 |
| | Substitute (3, –2, –1) and values of *a*, *b* and *c* in general equation and find *d* | M1 |
| | Obtain answer $2x - 2y + z = 9$ , or equivalent | A1 |
| | *OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ | (M1 |
| | Obtain two correct components | A1 |
| | Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ | A1 |
| | Substitute (3, –2, –1) in $-6x + 6y - 3z = d$ , or equivalent, and find *d* | M1 |
| | Obtain answer $-2x + 2y - z = -9$ , or equivalent | A1) |
| | *OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane | (M1 |
| | State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ | A1 |
| | State three correct equations in *x*, *y*, *z*, $\lambda$ and $\mu$ | A1 |
| | Eliminate $\lambda$ and $\mu$ | M1 |

| Question | Answer | | Marks |
|---|---|---|---|
| | | Obtain answer $2x - 2y + z = 9$, or equivalent | **A1)** |
| | *OR*3: | Using the relevant point and relevant vectors, form a determinant equation for the plane | **(M1** |
| | | State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$ | **A1** |
| | | Attempt to expand the determinant | **M1** |
| | | Obtain two correct cofactors | **A1** |
| | | Obtain answer $-2x + 2y - z = -9$, or equivalent | **A1)** |
| | | | **5** |