Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS	**9709/31**
Paper 3	**May/June 2018**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **11** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law for the logarithm of a product, quotient or power	**M1**
	Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	**A1**
	Solve for x	**M1**
	Obtain answer x = 1.52 only	**A1**
		4

Question	Answer	Marks
2(i)	Use trig formulae and obtain an equation in sin x and cos x	**M1***
	Obtain a correct equation in any form	**A1**
	Substitute exact trig ratios and obtain an expression for tan x	**M1(dep*)**
	Obtain answer $\tan x = \dfrac{-\left(6+\sqrt{6}\right)}{\left(6-\sqrt{2}\right)}$ or equivalent	**A1**
		4
2(ii)	State answer, e.g. 118.5°	**B1**
	State second answer, e.g. 298.5°	**B1ft**
		2

Question	Answer	Marks
3	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a \sin x = b$	**M1 ***
	Solve for x	**M1(dep*)**
	Obtain answer 0.340	**A1**
	Obtain second answer 2.802 and no other in the given interval	**A1**
		6

Question	Answer	Marks
4	*EITHER:* Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
	Obtain quotient $x^2 + 3x + 2$	**A1**
	Either Set remainder identically equal to zero and solve for *a* or for *b*, *or* multiply given divisor and found quotient and obtain *a* or *b*	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
	OR: Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in *B* and/or *C*	**M1**
	Obtain $B = 3$ and $A = 2$	**A1**
	Either Use equations to obtain *a* or *b* *or* multiply given divisor and found factor to obtain *a* or *b*	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
		5

Question	Answer	Marks
5(i)	State or imply $dx = -2\cos\theta\sin\theta\,\mathrm{d}\theta$, or equivalent	**B1**
	Substitute for x and dx, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
5(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

Question	Answer	Marks
6(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln x$	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms $a \ln x$ and $b\,t\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
6(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

Question	Answer	Marks
7(i)	Use quadratic formula, or completing the square, or the substitution $z = x + iy$ to find a root, using $i^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}i$	**A1**
		3
7(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
7(iii)	State or imply correct value of a relevant length or angle, e.g. *OA*, *OB*, *AB*, angle between *OA* or *OB* and the real axis	**B1ft**
	Carry out a complete method for finding angle *OAB*	**M1**
	Obtain *AOB* = 60° correctly	**A1**
		3
7(iv)	Give a complete justification of the given statement	**B1**
		1

Question	Answer	Marks
8(i)	Integrate by parts and reach $lxe^{-\frac{1}{2}x} + m\int e^{-\frac{1}{2}x}\,dx$	**M1***
	Obtain $-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}\,dx$	**A1**
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Having integrated twice, use limits and equate result to 2	**M1(dep*)**
	Obtain the given equation correctly	**A1**
		5
8(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
8(iii)	Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once	**M1**
	Obtain final answer 3.36	**A1**
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	**A1**
		3

Question	Answer	Marks
9(i)	State or imply the form $A + \frac{B}{x-1} + \frac{C}{3x+2}$	**B1**
	State or obtain $A = 4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B = 3$, $C = -1$	**A1**
	Obtain the other value	**A1**
		5
9(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\frac{1}{2} - \frac{9}{4}x - \frac{33}{8}x^2$	**A1**
		5

Question	Answer	Marks
10(a)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\,\mathbf{i}+(4+2\mu)\mathbf{j}+(4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu=-\frac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
	*OR*2: Calling (4, 2, 5) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+4\mathbf{j}+4\mathbf{k}$	**B1**
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	**M1**
	Obtain correct answer $21/\sqrt{14}$, or equivalent	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 1.22	**A1**
	*OR*3: State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	**B1**
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	**M1**
	Obtain correct answer, e.g. $4\mathbf{i}+\mathbf{j}-2\mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain answer 1.22	**A1**
		5

Question	Answer	Marks
10(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $a + 2b + 3c = 0$	**B1**
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	**M1**
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	**A1**
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathrm{k}$	**A1**
	Substitute a relevant point and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*2: Using a relevant point and relevant vectors form a 2-parameter equation for the plane	**M1**
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain correct answer $4x + y - 2z = 8$, or equivalent	**A1**
		5