Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6101943949

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Expand $\dfrac{4}{\sqrt{(4-3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

3 Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x\cos 3x\,\mathrm{d}x$, giving your answer in terms of π. [5]

4 The curve with equation $y = \dfrac{\ln x}{3 + x}$ has a stationary point at $x = p$.

(i) Show that p satisfies the equation $\ln x = 1 + \dfrac{3}{x}$. [3]

(ii) By sketching suitable graphs, show that the equation in part **(i)** has only one root. [2]

(iii) It is given that the equation in part **(i)** can be written in the form $x = \dfrac{3+x}{\ln x}$. Use an iterative formula based on this rearrangement to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 **(i)** By first expanding $\left(\cos^2 x + \sin^2 x\right)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \tfrac{3}{4}\sin^2 2x. \qquad [4]$$

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \tfrac{2}{3},$$

for $0° < x < 180°$. [4]

6 **(i)** Express $\dfrac{1}{4 - y^2}$ in partial fractions. [2]

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(ii) The variables x and y satisfy the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

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7 **Throughout this question the use of a calculator is not permitted.**

(i) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan\alpha$. [3]

(ii) Hence, showing all necessary working, show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos\theta + 2\sin\theta)^2}\,d\theta = 5.$ [5]

8 The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x^2 - y^2}{y^2 + 2xy}$. [4]

(ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

9 **(a)** Find the complex number z satisfying the equation

$$3z - iz^* = 1 + 5i,$$

where z^* denotes the complex conjugate of z. [4]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| \leqslant 3$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of z. Calculate the greatest value of $\arg z$ for points in this region. Give your answer in radians correct to 2 decimal places. [5]

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10 The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

The point P, with parameter t, lies on l and is such that angle PAB is equal to $120°$.

(ii) Show that $3t^2 + 8t + 4 = 0$. Hence find the position vector of P. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.