# Cambridge International A Level

**MATHEMATICS** **9709/32**

Paper 3 Pure Mathematics **March 2020**

MARK SCHEME

Maximum Mark: 75

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

| Mathematics-Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

| | |
|---|---|
| **M** | Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer. |
| **A** | Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied). |
| **B** | Mark for a correct result or statement independent of method marks. |
| **DM** or **DB** | When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given. |
| **FT** | Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. |

 

**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

 

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | Make a recognisable sketch graph of $y = \|x-2\|$ | **B1** | |
| | | **1** | |
| 1(b) | Find $x$-coordinate of intersection with $y = 3x - 4$ | **M1** | |
| | Obtain $x = \frac{3}{2}$ | **A1** | |
| | State final answer $x > \frac{3}{2}$ only | **A1** | |
| | **Alternative method for question 1(b)** | | |
| | Solve the linear inequality $3x - 4 > 2 - x$, or corresponding equation | **M1** | |
| | Obtain critical value $x = \frac{3}{2}$ | **A1** | |
| | State final answer $x > \frac{3}{2}$ only | **A1** | |
| | **Alternative method for question 1(b)** | | |
| | Solve the quadratic inequality $(x-2)^2 < (3x-4)^2$, or corresponding equation | **M1** | |
| | Obtain critical value $x = \frac{3}{2}$ | **A1** | |
| | State final answer $x > \frac{3}{2}$ only | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Use law of logarithm of a power and sum and remove logarithms | **M1** | |
| | Obtain a correct equation in any form, e.g. $3(2x+5)=(x+2)^2$ | **A1** | |
| | Use correct method to solve a 3-term quadratic, obtaining at least one root | **M1** | |
| | Obtain final answer $x = 1+2\sqrt{3}$ or $1 + \sqrt{12}$ only | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Sketch the graph $y = \sec x$ | **M1** | |
| | Sketch the graph $y = 2 - \frac{1}{2}x$, and justify the given statement | **A1** | |
| | | **2** | |
| 3(b) | Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$ | **M1** | |
| | Complete the argument correctly with correct calculated values | **A1** | |
| | | **2** | |
| 3(c) | Use the iterative formula correctly at least once | **M1** | |
| | Obtain final answer 0.88 | **A1** | |
| | Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Integrate by parts and reach $ax\tan x + b\int \tan x\,dx$ | **M1*** | |
| | Obtain $x\tan x - \int \tan x\,dx$ | **A1** | |
| | Complete the integration, obtaining a term $\pm\ln\cos x$, or equivalent | **M1** | |
| | Obtain integral $x\tan x + \ln\cos x$, or equivalent | **A1** | |
| | Substitute limits correctly, having integrated twice | **DM1** | |
| | Use a law of logarithms | **M1** | |
| | Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent | **A1** | |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Express LHS correctly as a single fraction | **B1** | |
| | Use $\cos(A \pm B)$ formula to simplify the numerator | **M1** | |
| | Use sin $2A$ formula to simplify the denominator | **M1** | |
| | Obtain the given result. | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | Obtain an equation in $\tan 2x$ and use correct method to solve for $x$ | **M1** | |
| | Obtain answer, e.g. 0.232 | **A1** | |
| | Obtain second answer, e.g. 1.80 | **A1** | Ignore answers outside the given interval. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Separate variables correctly and attempt integration of at least one side | **B1** | |
| | Obtain term of the form $a\tan^{-1}(2y)$ | **M1** | |
| | Obtain term $\frac{1}{2}\tan^{-1}(2y)$ | **A1** | |
| | Obtain term $-e^{-x}$ | **B1** | |
| | Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan^{-1}(by)$ and $ce^{\pm x}$ | **M1** | |
| | Obtain correct answer in any form | **A1** | |
| | Obtain final answer $y = \frac{1}{2}\tan\left(2e^{-1} - 2e^{-x}\right)$, or equivalent | **A1** | |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | State that $y$ approaches $\frac{1}{2}\tan\left(2e^{-1}\right)$, or equivalent | B1FT | The FT is on correct work on a solution containing $e^{-x}$. |
| | | 1 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | State or imply $3y^2 + 6xy\ \frac{dy}{dx}$ as derivative of $3xy^2$ | B1 | |
| | State or imply $3y^2\ \frac{dy}{dx}$ as derivative of $y^3$ | B1 | |
| | Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$ | M1 | Need to see $\frac{dy}{dx}$ factorised out prior to AG |
| | Obtain the given answer correctly | A1 | AG |
| | | 4 | |
| 7(b) | Equate denominator to zero | *M1 | |
| | Obtain $y = 2x$, or equivalent | A1 | |
| | Obtain an equation in $x$ or $y$ | DM1 | |
| | Obtain the point (1, 2) | A1 | |
| | State the point $\left(\sqrt[3]{5},0\right)$ | B1 | Alternatively (1.71, 0). |
| | | 5 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$ | B1 | |
| | Use a correct method to find $\overrightarrow{MN}$ | M1 | |
| | Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ | A1 | |
| | | 3 | |
| 8(b) | Use a correct method to form an equation for *MN* | M1 | |
| | Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent | A1 | |
| | | 2 | |
| 8(c) | Find $\overrightarrow{DP}$ for a point *P* on *MN* with parameter $\lambda$, e.g. $(2-\lambda, 1+2\lambda, -2+2\lambda)$ | B1 | |
| | Equate scalar product of $\overrightarrow{DP}$ and a direction vector for *MN* to zero and solve for $\lambda$ | M1 | |
| | Obtain $\lambda = \frac{4}{9}$ | A1 | |
| | State that the position vector of *P* is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$ | A1 | |
| | | 4 | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 9(a) | State or imply the form $\frac{A}{1+2x}+\frac{B}{1-2x}+\frac{C}{2+x}$ | **B1** | |
| | Use a correct method for finding a constant | **M1** | |
| | Obtain one of $A=-2$, $B=1$ and $C=4$ | **A1** | |
| | Obtain a second value | **A1** | |
| | Obtain the third value | **A1** | |
| | | **5** | |
| 9(b) | Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$ | **M1** | |
| | Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction | **A1FT + A1FT + A1FT** | The FT is on $A$, $B$ and $C$. |
| | Obtain final answer $1+5x-\frac{7}{2}x^2$ | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 10(a) | Solve for $v$ or $w$ | M1 | |
| | Use $i^2 = -1$ | M1 | |
| | Obtain $v = -\dfrac{2i}{1+i}$ or $w = \dfrac{5+7i}{-1+i}$ | A1 | |
| | Multiply numerator and denominator by the conjugate of the denominator | M1 | |
| | Obtain $v = -1 - i$ | A1 | |
| | Obtain $w = 1 - 6i$ | A1 | |
| | | 6 | |
| 10(b)(i) | Show a circle with centre 2 + 3i | B1 | |
| | Show a circle with radius 1 and centre not at the origin | B1 | |
| | | 2 | |
| 10(b)(ii) | Carry out a complete method for finding the least value of arg $z$ | M1 | |
| | Obtain answer $40.2°$ or 0.702 radians | A1 | |
| | | 2 | |