MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

The question paper contains a statement in the rubric on the front cover that states, 'no marks will be given for unsupported answers from a calculator.' This means that working must be shown to justify solutions of quadratic equations, trigonometric equations and simultaneous equations. In the case of quadratic equations, for example, it would be necessary to show factorisation or use of the formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down the solution alone is not sufficient; it is also not sufficient to quote the formula; values need to be shown substituted into it.

Knowledge of the values of trigonometric functions of angles expressed in both degrees and radians is most useful for this paper.

General comments

Several question parts on this paper involved substitution of given values into a formula or integral. Candidates should ensure they show clear evidence of the substitution, rather than just writing the final answer.

Comments on specific questions

Question 1

(a) This question was correctly answered by most candidates. It was clear, however, that some candidates had misunderstood the request for the powers to be in 'ascending' order. Some candidates didn't identify that one is a power of *x*.

(b) The use of $(-2x)$ in the expansion made this question challenging for some candidates, however, many correct answers were seen to this part.

(c) The selection of the three appropriate products commonly resulted in the correct answer for those candidates who had correct solutions to parts **(i)** and **(ii)**.

Question 2

Many candidates found this to be an accessible question and used suitable substitutions to reach a quadratic equation. Where a valid method (factorisation, formula or completion of the square) was used to solve the equation full marks were often gained. Candidates should however be reminded to ensure they show their method, rather than direct use of a calculator to solve the equation (as mentioned in the **Key messages**.

Candidates did not use a suitable substitution very occasionally reached the correct quartic equation but were often unable to proceed using a valid method.

Question 3

Correctly clearing the denominator and replacing $\tan\theta$ by $\dfrac{\sin\theta}{\cos\theta}$ was seen in many solutions. Some candidates chose to divide through by $\sin\theta$ without considering the solutions from $\sin\theta = 0$. A number of solutions incorrectly included $180 - \theta$ which did not gain the final accuracy mark.

Question 4

The method of forming a three term quadratic equation by elimination of *y* and setting the discriminant greater than zero was well understood and used by many to find the correct set of values. The use of a calculator to solve the quadratic in *k* resulted in at least one mark not being able to be awarded.

Question 5

(a) As a new topic to this examination fewer attempts were seen to this question.

For those that attempted this question, the transformations were often identified correctly but the descriptions were often incorrect. Those who identified the one way stretch parallel to the *y*-axis often didn't realise that this, rather than a translation, resulted in the increased displacement from the *y*-axis.

It should be noted that using descriptions involving up, down, along, across, backwards, forwards, horizontal, vertical without reference to the *x*- or *y*-axes is usually insufficient.

(b) Correct answers to part **(a)** were usually successfully incorporated into $y = f(x)$ to give the correct equation. Some candidates did not make the connection with their answers to part **(a)** and some were able to complete this part without completing part **(a)**.

Question 6

(a) The application of the chain rule was often seen. Where $\frac{dx}{dt}$ was identified as the required term the correct answer usually followed.

(b) The requirement to integrate the gradient equation was apparent to many candidates and they often went on to complete the integration correctly including the calculation of the constant of integration. To obtain full marks it was essential to complete the solution by fully stating the equation of the curve.

Question 7

(a) Most candidates were able to produce a correct completed square form of $f(x)$. The given domain appeared to confuse some candidates who chose the incorrect side of $f(x) = 2$ although the correct side was apparent from the completed square form.

(b) The process of finding the inverse from the completed square form was well understood and generally applied correctly. However, only when candidates understood the domain of $f(x)$ was the range of $f^{-1}(x)$ were they able to select the negative square root in the function to successfully complete the process.

(c) The formation of a quadratic equation in x from the combined function was seen in most candidates' answers. Some candidates did not show the solution of the resulting quadratic equation and others quoted the solution outside the domain of $f(x)$.

Question 8

(a) Consideration of the orientation of the three given points enabled candidates to deduce $\angle ABC = 90^o$ from which they were able to deduce that the mid-point of *AC* was the centre of the circle and then calculate the length of the radius. With this information the correct equation was usually stated.

Substitution of the three given coordinate pairs into the general equation of a circle was often chosen with less success because of algebraic errors.

(b) This was the most frequently omitted part of a question. Of those candidates who had found the centre in part **(a)** some went on to find the gradient of the radius to *B* and then used this to find the gradient and equation of the tangent at *B*. Those who assumed that *AC* was parallel to the tangent at *B* could not gain credit.

Question 9

(a) (i) Most candidates were able to write down the sum to infinity of a geometric progression and use the given information to find the common ratio. Aware of the answer, many went on to find the second term of the progression correctly.

(ii) The formula for the sum of a geometric progression was usually quoted correctly but the substitution in the common ratio expression at the given value of θ was not always clear. Those who gained full marks on this part showed their substitution and subsequent calculations clearly and rounded their answers appropriately.

(b) The calculation of the common difference was often presented correctly using the information given in the question. Most answers used the correct n^{th} term formula and, again, those who were able to then substitute the given value of θ usually reached the correct answer. Those who found the sum of the terms were able to gain some credit for finding the common difference.

Question 10

(a) The relationship between the sector and triangle areas was seen by most candidates and the area of the sector *ABC* was usually expressed correctly. The candidates who chose to find the area of triangle *ADE* directly from the formula '$A = \frac{1}{2}ab\sin C$' generally progressed to a correct equation and solution.

Those who attempted to use the height and base of triangle *ADE* were generally less successful and often lost accuracy in their final answers. The use of radians proved difficult for a minority of candidates.

(b) This part was the most frequently omitted and the generalisation $(ka)^2 \sin\theta = \frac{1}{2}a^2\theta$ was rarely seen. Candidates who were able to see this from their working in part **(a)** usually went on to find a suitable inequality but few completely correct solutions were seen because negative values of k were often then included.

Question 11

(a) In almost all answers to this part candidates appeared aware that $y = 0$ was the starting point. Correct answers from correct working were often seen. Answers unsupported by correct working gained no credit.

(b) Correct expressions for the gradient of the curve were often seen and many of these were used to obtain the gradient of the tangent. A minority elected to find the equation of the normal suggesting more attention should be given to reading the question carefully.

(c) Most attempts at this part involved candidates setting their gradient equation to zero and correct working often resulted in the required *x*-value. As in part **(a)** answers unsupported by working out gained no credit.

(d) Some very good answers were seen to this part. Most answers used the answer to part **(a)** as the lower limit for integration and most candidates appeared to realise that the substitution of the limits should be shown in their working.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

Candidates need to understand that parallel lines only require the consideration of the directional vectors of the lines, not the full vectors of the lines, and that parallel lines require the directional vector of one line to be a multiple of the directional vector of the other line, not for the directional vectors to be actually equal to each other, see **Question 7(a)**.

Candidates also need to be able to differentiate using the chain rule, which was needed for **Question 10(b).** Candidates are reminded that the calculator cannot be used in answering complex number questions. This was important for **Question 8(a)**.

Candidates need to understand how to apply the law of logarithms and indices. This was particularly key for **Question 1** and **Question 9(c)**.

General comments

The standard of work on this paper was high, with a considerable number of candidates performing well on many of the questions.

Candidates were generally aware of the need to show sufficient working in their solutions, especially in **Question 8(a)** and **Question 8(b)**. Candidates should consistently ensure that they include necessary detail in their solution, as solutions taken directly from a calculator are not sufficient.

Candidates should make sure their working is set out clearly in a logical manner and that all letters and symbols are of reasonable size and clearly distinguishable.

Comments on specific questions

Question 1

Most candidates were able to express $3\ln x$ as $\ln x^3$, convert $\ln x^3 - \ln 3$ to $\ln\left(\frac{x^3}{3}\right)$ and then obtain a correct equation free of logarithmic terms. However, a few candidates mistakenly gave their final answer correct to 3 decimal places instead of 3 significant figures, as demanded by the question.

Question 2

Most candidates successfully identified that this question was best approached using the remainder theorem rather than long division. Whilst long division is perfectly valid, the required arithmetic usually results in numerous errors. Most candidates correctly substituted for $x = -1$ and $x = -2$ or divided by the correct factors if using long division.

Question 3

This question was answered very well by most candidates. Only a few did not know how to expand $\tan(x + 45°)$, express $\cot x$ as $\frac{1}{(2\tan x)}$ or that $\tan 45° = 1$. However, many candidates believed $2\cot x$ should be expressed as $\frac{1}{(2\tan x)}$. In addition, in the algebra required to obtain a quadratic equation in terms of tan$x$ it was common to see the omission of the multiplication of some of the terms when completely removing the denominator. This, together with the error in the 2cot x term, often resulted in not being able to establish a 3-term quadratic equation. The solution of the candidates' quadratic equation was nearly always performed correctly for the angle in the first quadrant. A large number of candidates who had correct solutions for tan$x$ then opted for an incorrect second answer of (180° – 31.7°), as opposed to that from the negative value of x when converted into the domain $0° < x < 180°$. This was perhaps caused by them wrongly believing that since the angle was negative it could be rejected prior to relating it to the equivalent angle in the second quadrant.

Question 4

(a) Candidates nearly always separated the variables correctly and established ln y and $\pm\ln(1 - \cos x)$. Whilst the sign error in the integration of $\frac{(\sin x)}{(1-\cos x)}$ caused few issues, those who believed cos π to be equal to 1 soon found an inappropriate ln term appearing in their working.

(b) The majority of candidates found this question challenging. Few had a graph that passed through (0, 0), (2π, 0) and that was symmetrical about $x = \pi$.

Question 5

(a) Candidates generally found this part relatively straight forward, although some candidates made errors by not finding an exact value of R or giving α to 2 decimal places. If candidates choose not to use the trigonometrical expansions of $A \pm B$, and instead opt to quote results, such as $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$, then they need to be certain which coefficient corresponds to b and which to a.

(b) Most candidates successfully found $\beta = \sin^{-1}\left(\frac{1}{\sqrt{11}}\right)$. Some candidates did not introduce 2θ, continuing with x, and then struggled with the rest of the question. Other candidates reached $\theta = \frac{(\beta-\alpha)}{2}$, but finding it negative often simply rejected it. Others tried to convert the angle into the required interval but often finished with between one and three of the following answers: $2\theta = 180° \pm \beta - \alpha$ or $360° \pm \beta - \alpha$, the correct answers being $\theta = \frac{(180° - \beta - \alpha)}{2}$ or $\frac{(360° + \beta - \alpha)}{2}$.

Question 6

(a) Candidates found this partial fraction question to be more challenging, likely due to the presence of the constant a. Those who multiplied out and equated coefficients, or set $x = 3a$ and $\frac{a}{2}$, were usually successful, but those who set x to just a numerical value usually struggled to then correctly find the constants in their partial fractions. Some set a to a value, which was permitted provided they reverted back to a in their final answer.

(b) Most candidates realised that both these terms integrated to produce ln terms, however very few had $\ln(2x - a)$ and $-\ln(3a - x)$. Some candidates decided to use $\frac{-1}{(x-3a)}$ rather than $\frac{1}{(3a-x)}$ in order to simplify the integration. Whilst this is acceptable, candidates needed to introduce $\ln|x - 3a|$ to avoid lns with negative arguments when introducing limits.

Question 7

(a) Most candidates knew how to show that the two lines did not intersect, although occasionally they had an arithmetical error present. However, to show skewness requires an investigation of the directional vectors of the two lines, see **Key messages**. Few candidates were able to undertake this successfully, since they either used the position vectors or line vectors as opposed to the direction vectors, and related that they needed to not be equal to each other rather than not be a multiple of each other.

(b) The majority of candidates scored full marks on this question. Occasionally an arithmetical error was seen, but few compared with those that used the incorrect vectors; that is used line vectors or position vectors as opposed to direction vectors. Another error seen was when a different pair of vectors was used to establish the scalar product compared to that used to establish the moduli.

Question 8

(a) The majority of candidates knew to multiply the numerator and denominator by 3 – i, and showed all their working, see **Key messages**. However, many candidates had an arithmetical error in at least one of their operations.

(b) Evaluating the modulus was almost always performed correctly with working shown. However, this was not the case regarding the argument and correct answers were rarely seen. Candidates should always draw a sketch of the Argand diagram clearly showing the complex number for which they are trying to find the argument. The argument should be in radians, not degrees, and since an exact answer was requested, 135° and 2.36 both received no credit.

(c) Most candidates correctly stated that *OA* and *BC* were parallel. Many candidates stopped having found *OC* and sketched *OA* and *BC*. A better approach would have been to establish *BC* = –8 + 4i and compare with *OA*, hence stating *OA* and *BC* parallel and *BC* = 2*OA*.

(d) For this question, it was essential for candidates to show correct working with full details, as the answer was given. Stating that angle $AOB = \pi - \tan^{-1}\left(\frac{-1}{1}\right) - \tan^{-1}\left(\frac{1}{1}\right) = \frac{3\pi}{4}$ is incorrect working as $\tan^{-1}\left(\frac{-1}{1}\right)$ is itself $\arg(-1+\mathrm{i}) = \frac{3\pi}{4}$. Likewise, stating that angle $AOB == \pi - \tan^{-1}\left(\frac{1}{1}\right) = \frac{3\pi}{4}$ is insufficient as this does not relate angle *AOB* to arg(–1 + i). The correct approach was to relate angle *AOB* to $\arg u - \arg v = \arg\left(\frac{u}{v}\right)$, hence from **(b)** $= \frac{3\pi}{4}$. Stopping at arg*u* – arg*v* and using these individual arguments will not allow an exact answer. Other approaches are to find tan *AOB* from gradients of *OA* and *OB* and then use the tan(*A* – *B*) formula or to obtain cos *AOB* by using the cosine rule or a scalar product.

Question 9

(a) Most candidates evaluated $x - \mathrm{f}(x)$ or some modification of this, together with a sound argument, to correctly obtain both marks. However, those candidates who found the four values of x and f(x) at $x = 1$ and $x = 1.5$ struggled with this question, as they often omitted any comparison, or, if they did undertake one, they did it between the x values and between the f(x) values instead of between x =1 and f(1) and between 1.5 and f(1.5).

(b) This question was answered well by most candidates. Occasionally the answer of 1.199 was seen.

(c) Most candidates correctly used the quotient rule, although a few omitted the denominator or had one or more sign errors somewhere within the formula. The main challenge was establishing a correct quadratic equation due to the powers of the exponential terms and the algebra involved. Many candidates who managed to establish the correct quadratic equation usually finished with two solutions and did not reject the negative result. Answers were often expressed in decimal form instead of the exact form requested by the question.

Question 10

(a) This question was well answered by many candidates, although several did not establish a correct integral in terms of u. Errors ranged from replacing $\sin 2x$ by $\sin x$, $\cos^2 x$ incorrectly by $(1 - u)$ or $\sin 2x$ by $\sin x \cos x$. If candidates established the correct integral in u, they usually completed the question successfully, although occasionally the x limits were confused with the u limits.

(b) Most candidates knew how to proceed with this question but experienced problems applying the chain rule to $\sin 2x$ and to $\cos^2 x$. Whilst a substitution for $\sin 2x$ did help with the differentiation of $\sin x$, it did little to help with differentiating the now $\cos^3 x$. In addition, such substitution often led to other problems, for example the omission of the factor 2 from part of the expression when replacing $\sin 2x$ by $2\sin x\cos x$. It is essential that the initial differentiation is undertaken correctly otherwise it is impossible to establish a correct trigonometrical equation that candidates can solve.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

Non-exact numerical answers are required to be correct to 3 significant figures, as stated on the front cover rubric of the question paper. Cases where this was not adhered to were seen in **Question 2(b)**, **Question 7(a)** and **Question 7(b)**. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram would be particularly useful here in **Question 3**, **Question 5** and **Question 7**.

In questions such as **Question 6** in this paper, where velocity is given as a non-linear function of time, implying that acceleration is not constant, it is important to realise that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Questions 1**, **2(a)** and **3** were found to be the easiest questions whilst **Questions 5(b)**, **6(b)** and **7(b)** proved to be the most challenging.

In **Question 2(a)**, the angle α was given exactly as $\sin \alpha = 0.1$ It is not necessary to evaluate the angle in this case and problems such as this can often lead to exact answers and so any approximation of the angle can lead to a loss of accuracy.

One of the rubrics on this paper is to take $g = 10$ and the majority of candidates did follow this instruction. In some cases it was not possible to achieve a correct given answer unless this value was used, such as in **Question 2(a)**.

Comments on specific questions

Question 1

This question was completed successfully by many candidates. The definition of momentum was known by almost all candidates. However, an error that was frequently seen was not to consider the fact that the initial velocities and hence momentum of the two particles were in opposite directions. This often lead to a sign error in the momentum equation. The final answer was in some cases given as a negative value when in fact the question asked for the speed, which must be positive.

Question 2

(a) Most candidates made a very good attempt at this question. Almost all used the fact that since the car is travelling at constant speed then the driving force acting on the car is the sum of the resistance force and the component of the weight of the car. As the power of the car is given, the speed of the car can be found by applying the equation $P = Fv$ where P is the given power, F is the driving force and v is the required speed. Very few errors were seen in solutions to this problem. It is, however, a case where the given answer is only achievable by using the value of the acceleration due to gravity as $g = 10$.

(b) This question involved the application of Newton's second law since acceleration is required. The forces acting on the car in this situation are the same driving force as in **part (a)** and the same resistance, but a different component of the weight due to the change of angle of the hill. The combination of these three forces produces the required acceleration. Although many excellent attempts were seen, some candidates forgot to include the driving force in their solution. As the sine of the angle is not exact, this is a case where the answer must be given to 3 significant figures. A number of candidates produced very good solutions but then only gave their answer to 2 significant figures.

Question 3

This question was well answered by most candidates. There are several possible approaches to this problem but most candidates used the method of resolving the forces acting on the particle *Q* both horizontally and vertically. These two equations involved the two different tensions in strings *PQ* and *QR*. Once stated these equations can be solved simultaneously. It is important to make clear any notation used, particularly distinguishing which of the variables used represents which tension, and all working should be shown. An error seen was to mix sine and cosine when resolving. Some candidates wrongly stated that the tension was the same in both strings, but this method resulted in equations that where not compatible. An alternative approach which some candidates took was to use Lami's Theorem. This is perfectly acceptable.

Question 4

(a) This question simply involved recognising that the acceleration in the first 1.5 seconds is represented by the gradient of the line during this time period in the velocity-time graph. Almost all candidates found this correctly. An error made by a few candidates was to give the answer correct to only 2 significant figures.

(b) In this part of the question the most straightforward approach, which was that taken by almost all candidates, is to evaluate the area above the t-axis from $t = 0$ to $t = 7$ (distance travelled by the elevator on the upward journey) and equate this to the area below the t-axis between $t = 13$ and $t = 21.5$ (distance travelled by the elevator on the downward journey). This technique will produce an equation for V which can readily be solved. Most candidates made a good attempt at this question but some were confused by the sign of V and often gave their answer as a negative value. However $-V$ was shown on the v-axis and so the value of V is positive. When candidates considered these areas as being made up of several different shapes, errors seen included omitting some of these triangular or rectangular shapes. The best approach is to consider the shapes as two trapezia.

(c) In this part it is necessary to find the deceleration of the elevator by considering the gradient during the time period from $t = 6$ to $t = 7$, which is when the elevator is decelerating on its upward journey. Many candidates found this correctly but some thought that the deceleration was the negative of that found in **part (a)**. Another error seen was to find the deceleration on the final downward stage between $t = 20$ and $t = 21.5$. Once this deceleration is found it is necessary to apply Newton's second law to the elevator and the forces acting are the tension in the cable and the given weight of the elevator and its passengers. An error seen at this stage was to use an incorrect sign for the acceleration.

Question 5

(a) This question required the candidates to use the information given in the question, namely that the block travels 2 metres in 5 seconds. By using any of the relevant correct constant acceleration equations such as $s = ut + \frac{1}{2}at^2$, with $u = 0$, $s = 2$ and $t = 5$ the required acceleration can be found. Some candidates tried to find the velocity after 5 seconds but some made an error by assuming this was $v = \frac{2}{5}$, forgetting that the block is accelerating. However, most candidates found the correct value of the acceleration.

(b) The best method of approach to this question involves first resolving forces on the block in a direction perpendicular to the floor. Three forces are involved, namely the weight of the block, the normal reaction on the block and a component of the force X N. Some candidates forgot to include the effect of the component of X and so incorrectly thought that $R = 5g$. Newton's second law must be applied to the block in the direction of motion, where the forces which produce the acceleration found in **5(a)** are a component of X and the resistance force. As the block is moving, the resistance force, F, is given by $F = 0.4\,R$. Some candidates used the wrong component of X and some did not include the acceleration.

(c) In this part of the question there is no acceleration since the block is in limiting equilibrium, however the relationship $F = \mu R$ does apply here. Hence it is necessary to find the values of F and R in this new situation. Resolving forces acting on the block in the direction parallel to the floor gives $F = 25 \cos 30$ and resolving forces on the block perpendicular to the floor gives $R = 5g - 25 \sin 30$. Substituting these values into the equation $F = \mu R$ gives the required value of the coefficient of friction, μ. Most candidates found the correct value of F however some candidates did not include the effect of the X component when determining R and others continued to include the effect of acceleration.

Question 6

(a) This question involves an expression for velocity which is a non-linear function of t and so this means that the constant acceleration equations cannot be used to solve this problem. Almost all candidates realised this and used a calculus based approach. Since the question asks for the displacement, integration of the expression for v is required here. Almost all candidates made a successful attempt at this part of the question. It was then necessary to use the limits of $t = 0$ and $t = 1$ to find the required displacement. The few errors seen were mainly in the integration of the term involving the fractional power and some numerical errors when dealing with the limits.

(b) This part of the question asks for the minimum velocity and most candidates realised that this happens when the acceleration is zero. In order to find the acceleration, a, it is necessary to differentiate the given expression for v. The minimum value of v occurs when $a = 0$. Most candidates made an attempt at this and produced an equation involving t, $\sqrt{t}$ and a constant term. By considering the equation as a quadratic in $\sqrt{t}$ it is possible to solve this equation and find the two values of t at which the maximum and minimum values of v occur. Again, most candidates followed this approach and many successfully found the two values of t. At this stage many candidates evaluated v at these two time values but this is not enough to show that the value found is a minimum. One method of showing this is to use an expression for $\mathrm{d}a/\mathrm{d}t$ and test the values of t and if $\mathrm{d}a/\mathrm{d}t > 0$ then the value found for v at this time is a minimum. Some excellent solutions were seen but most candidates finished their solution without checking that the value of v found was indeed a minimum.

Question 7

(a) This question involves connected particles P and Q. The motion is given as one in which P moves down the plane. In order to find the required value of the tension, T, in the string, Newton's second law must be applied to P and to Q. Alternatively Newton's second law could be applied to either one of the particles and also to the system. This was the approach taken by most candidates. Whichever method is used the two equations involve the acceleration, a, of the particles and the required tension, T. Errors seen when using this approach were to use the wrong components of the weights when resolving in directions parallel to the planes, omitting some of the force terms and in some cases not including the effect of the acceleration. Many candidates produced excellent solutions.

(b) Candidates found this question to be particularly challenging. It is stated in the question that energy methods must be used and so any method which only relied on a non-energy approach did not score marks. There are several possible approaches that can be taken to solve this problem. The energy method could be applied to the whole system, or alternatively, the energy method could be applied to either particle. However, this method of approach would involve finding the tension in the string. Since the given situation is different to that in **part (a)** the tension in this case is not the same as the tension found in **part (a)**. Many candidates who considered P or Q did then incorrectly attempt to use the value of T from **part (a)**.

In the system approach, the kinetic energy gained by each particle must be found. The change in potential energy must also be considered as P moves down the plane and Q moves up the plane both by 1 m. It is also necessary to include the effects of both the work done against friction and the work done by the 0.8 N driving force. The work-energy equation takes the form WD by 0.8 N + PE loss = KE gain + WD against friction. Errors made by candidates using this method included the omission of some terms, wrong signs when considering potential energy and using the distance travelled of 1 m as the change in height rather than a component of this. There were some very good answers seen using this method, but most candidates did not complete this question successfully.

Candidates considering the two particles separately must first find the tension, T, in the string by applying the work-energy equation to particle P in the form PE loss by P + WD by 0.8 N = KE gain by P + WD by T. This enables the tension in the string to be found as $T = 3.21$ N. The work-energy equation must now be applied to particle Q whose mass is to be found. The work-energy equation here takes the form of WD by tension = PE gain by Q + KE gain by Q + WD against friction. Very few candidates successfully used this method. The most common error seen in this method was to incorrectly use the tension found in **part (a)** or to not consider the effect of tension at all.

MATHEMATICS

Paper 9709/52
Probability and Statistics 1

Key messages

Candidates should be aware of the need to communicate their methods clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. When errors are corrected, candidates would be well advised to cross through and replace the term.

Candidates should only state non-exact answers to 3 significant figures, exact answers should be stated exactly. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures answer. The only exception is if a value is stated within the question. There is no requirement for probabilities to be stated as a decimal and it is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Many candidates labelled theircumulative frequency graphs clearly.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates did not complete the very last question. A few candidates struggled with some topics, in particular the normal distribution. Many good solutions were seen for **Questions 2** and **4**. The context in **Questions 1** and **7** was found to be challenging for many.

Comments on specific questions

Question 1

The majority of candidates identified that the geometric distribution was the most appropriate approach in both parts of the question. Candidates should be aware that there is no requirement to convert an exact rational answer to a rounded decimal, which is less accurate.

(a) Good solutions included a clear statement of the calculation being evaluated. Some weaker solutions simply stated a value, which is inappropriate. A large number of final answers were stated to 2 significant figures, which possibly indicates difference misunderstanding between decimal places and significant figures. A common error was to simply calculate the probability of not obtaining a score of 3 seven times.

(b) The most efficient method was to calculate the complement of not scoring 3 five times using a geometric approximation. Many candidates simply summed the probabilities of scoring a 3 for the five possible outcomes, and occasionally the formula for summing a geometric series was used.

The alternative binomial distribution approach was attempted by a few candidates. The omission of the binomial coefficient was noted in many solutions. The incorrect inclusion of the sixth trial was frequent and candidates do need to be confident in interpreting the success conditions provided.

A small number of solutions assumed that the value found in **part (a)** was the probability required in the calculation. This highlights the good practice of re-reading the question after completing the solution to confirm that the requirements have been fulfilled appropriately.

Question 2

A large number of fully correct answers were seen. The best solutions used a tree diagram in **part (a)** to clarify the information provided and identified clearly that if a red scarf was worn, then the probability of wearing a hat was 1.

(a) Most solutions involved summing the three terms linked to the different colour scarves. Some candidates assumed that the individual probabilities for wearing a hat would sum to 1, so incorrectly determined that the probability of wearing a hat with a red scarf as 0.3 . The use of a tree diagram may have helped clarify the data provided. A small, but significant, number of correct expressions were inaccurately evaluated.

(b) All but the weakest solutions used a conditional probability formula approach to this question. The most efficient solutions recognised that the denominator was the complement of **part (a)**, but many candidates simply recalculated the required probability, which allowed any error in interpreting **part (a)** to be rectified. A number of correct expressions did not lead to full credit as the value found was truncated to 3 decimal places rather than rounded to 3 significant figures.

Question 3

Many good solutions were seen to this question and an efficient use of the provided normal distribution tables, or alternatives, was noted. The best solutions often had a simple diagram to clarify the conditions required in each part.

(a) The best solutions identified the required probability area and used the normal standardisation formula accurately twice. The majority recognised that the data was continuous, so no continuity correction was required. A variety of different methods to calculate the final probability were seen, the most common being simply to calculate $P(X < 100) - P(X < 85)$. An error seen in weaker solutions was assuming that $1 - P(X > 85)$ was equivalent to subtracting the associated z-value from 1.

Candidates should be aware that z-values should not be truncated, or rounded to less than 3 decimal places, before using the normal distribution tables.

(b) In the best solutions, a simple diagram identified that a final answer less than 96 was expected. Good solutions determined an appropriate negative z-value, formed an equation with the normal standardisation formula and solved efficiently with appropriate supporting manipulations. The weakest solutions simply equated a probability with the normal standardisation formula and could gain no credit.

Question 4

(a) Almost all candidates formed the expected a probability distribution table with all the anticipated outcomes. A small number of candidates found the value of k at this stage and did not follow the instructions in the question that their answer had to be in terms of k.

(b) Many fully correct solutions were seen. The best solutions provided clear evidence of the use of the sum of probabilities in **part (a)** to find k with clear expressions for the mean and variance calculations stated and evaluated accurately.

A significant number of solutions produced the variance and mean expressions initially in terms of k and then substituted and evaluated with their k value.

Question 5

Almost all candidates attempted at least part of this question. Many solutions did not treat the data as continuous, which resulted in a poor interpretation of the class groups stated in the table.

(a) Almost all candidates attempted to construct a cumulative frequency graph. The weakest solutions were often of a simple frequency graph, but a few histograms were also seen.

Where the axes were labelled, all the required information was provided including the units for the distance.

The best graphs had clear points identified at the upper boundary of each class with a smooth curve used. Some more able candidates used ruled lines and formed a cumulative frequency polygon which could not gain full credit.

The most common error was to plot the points at the higher value stated for each class, which ignored the continuous nature of the data. A few good solutions used the continuity adjustment for all but the final class.

Candidates should be aware that cumulative frequency graphs should have a curve which starts at a value of zero on the y-axis.

(b) Many candidates found this part challenging. Good solutions clearly stated that if 30 per cent of journeys are more than d km then 70 per cent are less. A calculation of 70 per cent of the journey total was shown with a clear reading from the graph being indicated by appropriate lines.

The most common error was to simply calculate 30 per cent of the journeys and use the graph at this value to find the distance travelled.

Candidates should ensure that when the questions states to 'Use your graph' that clear markings are shown on the graph to indicate how it has been used.

A noticeable number of solutions used 160 in their calculations, which seemed to link to the cumulative frequency axis being annotated to that value, rather than a miscalculation.

(c) There was much evidence that candidates had a good understanding of the process required to calculate the mean of grouped data. Good solutions included a full unsimplified expression for the mean, which was then efficiently evaluated using a calculator. However, many solutions did not recognise the continuous nature of the data, with the mid-point of the first interval being frequently mis-stated as 2, or more rarely as 2.5. Although their final value rounded to the anticipated answer, full credit would not be awarded because of the error in process.

Question 6

Almost all candidates identified whether the permutation or combination approach was appropriate in each part. Many candidates used of simple diagrams and clear explanations of their approach that was being attempted throughout this question.

(a) A large number of fully correct answers were seen. The best solutions included a clear unsimplified factor expression with the denominator removing the effect of all repeated letters. A few solutions did not identify that there were three different letters repeated. A small number of correct expressions were evaluated inaccurately.

Candidates should be strongly advised not to round exact values to 3 significant figures.

(b) The use of a simple diagram to interpret the required condition was often seen in good solutions.

Complete solutions were seen using both common approaches, with success being approximately equal.

The best subtraction approach solutions identified separately the total number of ways the letters can be arranged with the Rs at the end and then with the Rs at the end and the As together, assuming that they are a single item. The difference was then found accurately. A common error was to not remove the effect of the repeated letters in the calculations.

The best insertion approach solutions calculated the number of ways the letters excluding the Rs and As can be arranged and then stating that the As can be inserted in $^8C_2$ ways and multiplying. Common errors were not removing the effect of the repeated Ls or assuming that the order of the As was important and using $^8P_2$.

Weaker solutions often ignored that the Rs must be at the end and used all the letters at the start of their method.

(c) This question was found challenging by many candidates. Solutions which included a simple diagram to clarify the possible outcome conditions were often more successful. The best solutions identified in a logical order the four possible scenarios, linked an appropriate calculation to each scenario and then summed to find the total number of selections possible. Although the majority of solutions only identified appropriate scenarios, additional incorrect scenarios were also seen regularly. The omission of RRAALL was not uncommon.

Solutions from mid-range candidates often implied that the As and Ls were not identical and introduced additional multiples of 2 when evaluating the number of possible outcomes for the anticipated scenarios.

Because of the required condition, an alternative approach of having RRAL as a fixed group and then finding the number of selections possible for the final two letters from the remaining letters of CATERPILLAR was legitimate. Candidates who used this approach were often successful.

Question 7

(a)(i) Almost all candidates calculated the required probability. The most common error was not stating the exact answer. Candidates should be aware that only non-exact answers should be rounded to 3 significant figures.

(ii) Although most candidates provided some evidence of understanding what was required to determine if the events were independent, many solutions did not include sufficient rigour to justify the conclusion. Good solutions stated an appropriate independence condition, used appropriate mathematical notation to identify expressions, identified the required probabilities and made a numerical comparison before stating a conclusion. A number of solutions used *A* and *B* without defining what was represented.

The weakest solution considered only the males and used P(males ∩ swimming) = $\frac{31}{180}$.

(b)(i) Almost all candidates recognised that the binomial distribution was the appropriate approach. The best solutions stated 1 – P(0,1,2) before starting any calculations. Good solutions provided a full, unsimplified expression which was evaluated efficiently. Weaker solutions often misinterpreted the required condition and included P(3) in the expression. Candidates should be aware that they must calculate to greater accuracy than the 3 significant figures expected for the final answer. Premature approximation or truncation during evaluation was identified in many solutions, which resulted in the final answer not being acceptable.

(ii) When attempted, most candidates recognised that the normal distribution was an appropriate approximation for the context. The best solutions provided clear justification for the mean and variance values, used the appropriate continuity correction for the discrete data, substituted values clearly in the normal standardisation formula, included a simple sketch to identify the required probability area and found the probability accurately from the *z*-value. Weaker solutions often omitted the continuity correction, or misinterpreted the boundary condition and used 32·5.

MATHEMATICS

Paper 9709/62
Probability and Statistics 2

Key messages

Numerical answers must have all relevant working clearly shown.

Final answers need to be given to at least a 3 significant figure accuracy, therefore accuracy of at least 4 significant figures should be maintained throughout all working.

Candidates should always check the sensibility of their answers within the context of the question, to help highlight any potential mistakes.

When writing conclusions to hypotheses tests the answer must be given in context and there must be a level of uncertainty.

Candidates need to be able to recognise a binomial distribution from a given scenario and correctly choose a valid approximating distribution when required.

General comments

There were some very good responses seen, as well as some weaker ones. Questions that proved more challenging for many candidates included **Questions 1(b)**, **3(b)** and **6**, whilst **Questions 1(a)**, **2(a)** and **4(c)** were generally well attempted.

Candidates did not appear to have a problem with timing on this paper.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some very good and complete solutions.

Comments on specific questions

Question 1

Many candidates made a good attempt at this question. The mean and the unbiased variance were calculated correctly by many candidates, though there were a few cases where the biased variance was mistakenly calculated. The z value used was mostly correct and many candidates used an expression of the correct form for the confidence interval. Many candidates did not get a final answer correct to 3 significant figures due to prematurely rounding their answer for the mean, with 80.3 used rather than 80.33 in their confidence interval calculation. It is important for candidates to note that if a final answer is required to 3 significant figure accuracy then all figures used in any calculations must be accurate to at least 4 significant figures. Most candidates gave their final answer as an interval, as required by the question.

Part (b) proved challenging for candidates. Many candidates gave answers which were not fully accurate; the distribution referred to must be unambiguously the population distribution.

Question 2

For candidates who were able to find f(x), this question was reasonably well attempted. In **part (a)** many candidates realised that the area under the line was equal to 1 and set up a correct equation to show that k was equal to 2. There were a few candidates who found k = 2 by incorrect methods, and a few candidates who successfully found $k = 2$ but used a long, though correct, method involving the equation of the line in the form $y = -0.5x + 0.5k$. Such correct methods, whilst gaining the marks, likely incurred a time penalty for these candidates.

The equation of the line was required for **part (b)**; common errors included the use of a positive gradient rather than a negative one, using an equation of a line with $c = 0$ (rather than the correct value $c = 1$) and on occasion incorrectly using an equation of a curve. It is important for candidates to check the sensibility of their answers against the information given; the line shown on the diagram has a negative gradient and does not go through the origin. Many candidates correctly integrated $x\text{f}(x)$ using their $\text{f}(x)$ and those with a correct $\text{f}(x)$ were usually successful in reaching the correct answer.

In **part (c)** the correct method was often used and marks were gained, depending on the accuracy of their $\text{f}(x)$ found in **part (b)**. A common error in the method used was to integrate with reversed limits. Many candidates reached a quadratic equation and most successfully rearranged to the correct form to solve and realised that one solution could be rejected when considering the context of the question.

Question 3

Part (a) was well attempted, with most candidates stating that a one-tail test was required and successfully explaining why. **Part (b)** was not as well attempted. Few candidates were successful in stating the hypotheses and many candidates made no attempt. The comparison was usually stated correctly, but the conclusion often lacked context or the language used by the candidate was not of the required level of uncertainty in that many candidates omitted a statement such as 'there is evidence to suggest…' or similar.

Question 4

Candidates found it a challenge to pick out a binomial distribution from the information given and to decide on a valid approximating distribution. As such, **parts (a)** and **(b)** were found particularly challenging by candidates. Despite this, many candidates went on to use the correct (Poisson) distribution in **part (c)** and follow through marks were available for those who stated an incorrect distribution in **part (b)**, though there were some candidates who did not use the distribution they stated. **Part (d)** was less well attempted; many candidates incorrectly used a normal distribution rather than Po(1.75), and some used B(700,0.0025) rather than finding an approximating distribution, hence not answering the question as set.

There were a few occasions when candidates gave unsupported final answers; it is important that all relevant working is shown so that the method used is clear.

Question 5

This question was well attempted by most candidates, with some candidates scoring very well. Errors included incorrect values for the variance in both **parts (a)** and **(b)**, often by confusing methods. In **part (a)** the variance was often incorrectly calculated as $3^2 \times 0.0102 + 4^2 \times 0.0036$ and in **part (b)** incorrectly calculated as $0.0102 + 2 \times 0.0036$. It is important that candidates can correctly distinguish between a sum of normal random variables and a multiple of a normal random variable.

Other errors included finding the wrong probability area. It is good practice for candidates to clearly state which probability they are finding (for example in **part (a)** $P(< 25.5)$ was required).The use of a diagram can often prevent an incorrect probability area being chosen.

Question 6

Candidates found this question challenging. There were occasions on this question when unsupported answers were given, and it is important that all relevant working is shown to fully support the answer reached.

In **part (a)** many candidates stated correct hypotheses, though on occasions μ was incorrectly used. It was required to find $P(X \geqslant 4)$, and common errors included finding $P(X = 4)$ or $P(X > 4)$. The distribution was B(25,0.08) and all terms in the Binomial expression needed to be clearly and fully stated. Some candidates used an incorrect distribution and some did not show the full Binomial expression. The next step was to show a clear comparison of the probability calculated with 0.05 in order to make a conclusion to the test, and this comparison was clearly stated by the majority of candidates. The comparison then justified the final conclusion to the test which needed to have been written in the context of the question, with language that used a level of uncertainty, for example using phrases such as ‘There is no evidence to suggest….’ or similar.

In **part (b)** the answer reached depended on the candidates’ conclusion to **part (a)**, and many candidates correctly chose and justified their answer.

Few candidates gained full marks in **part (c)** as answers were often not fully justified. Candidates that successfully calculated $P(X \geqslant 5)$ did not always show that this was less than 0.05. Again, the Binomial expression used to calculate the probability of $P(X \geqslant 5)$ should be clearly shown; not all answers were fully supported in this way.