Cambridge International A Level

MATHEMATICS **9709/41**

Paper 4 Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Force exerted by winch $= 50g\sin 60 + 100[= 433.0 + 100 = 533.0]$	**M1**	For resolving forces along the plane
	Work done $= 5\times(50g\sin 60 + 100)$	**M1**	Use of WD = Force × distance
	Work done = 2670 J	**A1**	
	Alternative method for Question 1		
	PE increase $= 50g \times 5\sin 60$	**M1**	Correct dimensions
	Work done $= 50g\times 5\sin 60 + 100\times 5$	**M1**	Apply the work-energy equation, 3 terms
	Work done = 2670 J	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	0.1 kg particle $T - 0.1g = 0.1a$ *m* kg particle $mg - T = ma$ System $mg - 0.1g = (m + 0.1)a$	**M1**	Apply Newton's 2nd law to either the 0.1 kg particle, the *m* kg particle or to the system, correct number of terms
		A1	Two correct equations
	Solve for *m* $[a = 5]$	**M1**	From 2 equations with the correct number of relevant terms
	$m = 0.3$	**A1**	
		4	

Question	Answer	Marks	Guidance
2(b)	$v^2 = 0 + 2 \times 5 \times 0.9$	**M1**	Use of $v^2 = u^2 + 2as$ with $u = 0$, $s = 0.9$ and *their* $a \neq \pm g$
	$v = 3$ m s^{-1}	**A1 FT**	FT on $\sqrt{1.8a}$
		2	

Question	Answer	Marks	Guidance
3(a)	Use of conservation of momentum, 3 terms	**M1**	Correct dimensions
	$0.1 \times 5 + 0 = 0.1 \times (-1) + 0.2 \times (\pm v)$	**A1**	
	$v = 3$ m s^{-1}	**A1**	A0 for $v = -3$
		3	
3(b)	$0.2 \times \textit{their}\, 3 + 0 = 0.2 \times u + 0.5V$	**M1**	Use of conservation of momentum, 3 terms, correct dimensions. Allow $u = 0$ used or if Q and R coalesce
	$u \geqslant -1$	**B1**	Allow $u = -1$. Allow equality for finding greatest value of V. Condition for no collision with P, may be a statement.
	Greatest $V = 1.6$	**A1 FT**	FT on *their* 3 from **3(a)** if $u = -1$ used.
		3	

Question	Answer	Marks	Guidance
4(a)	Isabella $v = 5 \times 1.1 \; [= 5.5]$	**B1**	Isabella's constant speed for 10 seconds
	Use of $s = ut + ½at^2$ or use of v–t graph to find total distance	**M1**	For either Isabella or Maria, all sections included but allow one error in use of formulae
	$s_I = \frac{1}{2} \times 1.1 \times 5^2 + 10 \times 5.5 + \frac{1}{2} \times 1.1 \times 5^2 \; [= 82.5]$ or $s_I = \frac{1}{2} \times (20 + 10) \times 5.5 \; [= 82.5]$	**A1**	For correct expression for Isabella, accept unsimplified
	$s_M = 27.5 + 5 \times 10 + \frac{1}{2} \times 5 \times 5 \; [= 90]$	**A1**	For correct expression for Maria, accept unsimplified
	Distances for Isabella = 82.5 and Maria = 90, so Maria goes further	**B1**	
		5	
4(b)	$\frac{1}{2} a \times 5^2 + 10 \times 5a + \frac{1}{2} a \times 5^2 = 90$ or $\frac{1}{2} \times (20 + 10) \times 5a = 90$	**M1**	Attempt total distance travelled by Isabella and set up an equation for a, using their value of $s_M = 90$. All parts included, allow one error.
	$a = 1.2$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$v = \int \left(6t^{\frac{1}{2}} - 2t\right) \mathrm{d}t$	**M1**	For integration. $v = at$ is M0.
	$v = 4t^{\frac{3}{2}} - t^2 \; (+c)$	**A1**	Allow unsimplified coefficients.
	$v = 0$ leading to $t = 0$ or $t^{\frac{1}{2}} = 4$ leading to $t = 16$	**A1**	
		3	
5(b)	$6t^{\frac{1}{2}} - 2t = 0$	**M1**	Attempt to solve $a = 0$, using valid algebra, reaching $t = \ldots$
	$t = 9$	**A1**	
	$s = \int \left(4t^{\frac{3}{2}} - t^2\right) \mathrm{d}t$ $\left[s = \frac{8}{5}t^{\frac{5}{2}} - \frac{1}{3}t^3 \; (+c)\right]$	**M1**	For integration of their expression for v which includes a term with a fractional power. Allow unsimplified coefficients. $v = at$ is M0
	$s = \frac{8}{5}t^{\frac{5}{2}} - \frac{1}{3}t^3$	**A1**	For correct integral
	Distance = 145.8 m	**B1**	Allow $\frac{729}{5}$ or 146 to 3s.f.
		5	

Question	Answer	Marks	Guidance
6(a)	$20\cos 30 = 25\cos 60 + 10\cos\alpha$ $[17.32 = 12.5 + 10\cos\alpha, \ \rightarrow \cos\alpha = 0.4821]$	**M1**	For resolving forces horizontally, all relevant terms included
	$\alpha = 61.2$	**A1**	From $\alpha = 61.18$
	Resultant $= 20\sin 30 + 10\sin 61.2 - 25\sin 60$ $[= 10 + 8.761 - 21.651]$	**M1**	For resolving forces vertically, all relevant terms included
	Magnitude of resultant force = 2.89 N	**A1**	A0 for –2.89 N or for ±2.89 N. Allow 2.89 N downwards
		4	
6(b)	$X = 25\cos 60 + 10\cos 45 - 20\cos 30$ $= 12.5 + 7.07107 - 17.32051 = 2.25056$ $Y = 20\sin 30 + 10\sin 45 - 25\sin 60$ $= 10 + 7.07107 - 21.65064 = -4.57957$	**M1**	For either horizontal or vertical component, correct number of relevant terms. Allow $\pm X$ and/or $\pm Y$
		A1	For both correct, allow unsimplified
	$R = \sqrt{X^2 + Y^2}$	**M1**	OE. Using a method to find the resultant force, using expressions for X and Y with at least 5 relevant terms.
	$\alpha = \tan^{-1}\frac{Y}{X}$	**M1**	OE. A method to find the direction, using expressions for X and Y with at least 5 relevant terms.
	Resultant = 5.10 N, Direction = 63.8° below positive x-axis	**A1**	For both correct, angle clearly explained. May use a diagram with a correct arrow and arc for angle. Allow angle 296° (measured anticlockwise from +ve x-axis)
		5	

Question	Answer	Marks	Guidance
7(a)(i)	PE $= 35g \times 2.5 \sin 30$	**M1**	
	$\frac{1}{2} \times 35v^2 = 35g \times 2.5 \sin 30$	**M1**	Use of conservation of energy, 2 terms, correct dimensions
	$v = 5 \text{ m s}^{-1}$	**A1**	
	Alternative method for Question 7(a)(i)		
	$mg \sin 30 = ma$ leading to $a = 5$	**M1**	For applying Newton's 2nd law down the plane, 2 terms, correct dimensions
	$v^2 = 0 + 2 \times 5 \times 2.5$	**M1**	For using $v^2 = u^2 + 2as$, using *their* $a \neq \pm g$
	$v = 5 \text{ m s}^{-1}$	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)(ii)	$\frac{1}{2}\times 35\times 5^2 = 250d$	**M1**	Use of work-energy from the bottom of the slide until motion stops, 2 terms, correct dimensions, using *their* v
	$d = 1.75$ m	**A1**	
	Alternative method for Question 7(a)(ii)		
	$35g\times 2.5\sin 30 = 250d$	**M1**	Use of work-energy from the start until motion stops, 2 terms, correct dimensions.
	$d = 1.75$ m	**A1**	
	Alternative method for Question 7(a)(ii)		
	$-250 = 35a$ leading to $a = -\frac{50}{7} = -7.14$ $0 = 5^2 + 2(a)d$	**M1**	Newton's 2nd law on the horizontal section with resistance = 250 N to find a and use $v^2 = u^2 + 2as$ with $v = 0$, $u = 5$ and $s = d$.
	$d = 1.75$m	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	$\frac{1}{2}\times 35v^2 = 250\times 1.05 \; [v^2 = 15]$ or $-250 = 35a$ leading to $a = -\frac{50}{7}$ $0 = v^2 + 2\times -\frac{50}{7}\times 1.05 \quad [v^2 = 15]$	**B1**	Either use the correct work energy equation for motion on the horizontal section or use the fact that the frictional force on the horizontal section is 250 N in order to set up an equation that would lead to finding the speed at the bottom of the slide.
	$R = 35g\cos 30 [= 303.11]$	**B1**	
	$v^2 = 0 + 2\times a\times 2.5 = 15$ leading to $a = 3$ or PE change $= 35g\times 2.5\sin 30 [= 437.5]$	**M1**	For using $v^2 = u^2 + 2as$, with their v^2 to set up an equation that would lead to finding a.
	$35g\sin 30 - F = 35a$ or $[175 - F = 35a]$ or $35g\times 2.5\sin 30 = F\times 2.5 + \frac{1}{2}\times 35\times 15 \; [437.5 = F\times 2.5 + 262.5]$	**M1**	For using Newton's 2nd law down the slope with correct dimensions. or For using energy equation, 3 relevant terms with correct dimensions.
	$F = \mu\times R$	**M1**	For using $F = \mu R$, where R is a component of $35g$.
	$\mu = 0.231$	**A1**	Allow $\mu = \frac{2\sqrt{3}}{15}$ OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for Question 7(b)**		
	$R = 35g\cos 30$	**B1**	
	PE change $= 35g \times 2.5\sin 30 [= 437.5]$	**B1**	
	WD against friction on the flat $= 250 \times 1.05$	**B1**	WD = 262.5
	$35g \times 2.5\sin 30 = F \times 2.5 + 250 \times 1.05$ $[437.5 = F \times 2.5 + 262.5]$	**M1**	For using energy equation, 3 relevant terms with correct dimensions.
	$F = \mu \times R$	**M1**	For using $F = \mu R$ at any stage, where R is a component of $35g$.
	$\mu = 0.231$	**A1**	Allow $\mu = \frac{2\sqrt{3}}{15}$ OE
		6	