Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular equation $4^2\left(5^x-1\right)^2=\left(5^x\right)^2$ or pair of equations $4\left(5^x-1\right)=\pm 5^x$	**M1**	
	Obtain $5^x=\frac{4}{3}$ and $5^x=\frac{4}{5}$ (or $5^{x+1}=4$)	**A1**	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	**M1**	
	Obtain answers $x = 0.179$ and $x = -0.139$	**A1**	
	Alternative method for question 1		
	Obtain $5^x=\frac{4}{3}$ by solving an equation	**B1**	
	Obtain $5^x=\frac{4}{5}$ (or $5^{x+1}=4$) by solving an equation	**B1**	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	**M1**	
	Obtain answers $x = 0.179$ and $x = -0.139$	**A1**	
		4	

Question	Answer	Marks	Guidance
2(a)	State $R = \sqrt{34}$	**B1**	
	Use trig formulae to find α	**M1**	$\tan\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{3}{\sqrt{34}}$ or $\cos\alpha = \frac{5}{\sqrt{34}}$.
	Obtain $\alpha = 0.54$	**A1**	$30.96°$ scores **M1A0**.
		3	
2(b)	State greatest value 34	**B1 FT**	*Their* R^2 .
	State least value 0	**B1**	
		2	

Question	Answer	Marks	Guidance
3(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx} = e^{1-2x} - 2xe^{1-2x}$
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \frac{1}{2}$ and $y = \frac{1}{2}$	**A1**	
		4	
3(b)	Use a correct method for determining the nature of a stationary point	**M1**	e.g. $\frac{d^2y}{dx^2} = -2e^{1-2x} - 2(1-2x)e^{1-2x}$
	Show that it is a maximum point	**A1**	
		2	

Question	Answer	Marks	Guidance
4	State that $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{1}{2\sqrt{x}}\ dx$	**B1**	
	Substitute throughout for x and dx	**M1**	
	Obtain a correct integral with integrand $\frac{2}{u^2+1}$	**A1**	
	Integrate and obtain term of the form $k\tan^{-1}u$	**M1**	$\left(2\tan^{-1}u\right)$
	Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	**A1**	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
	Obtain answer $\frac{1}{3}\pi$	**A1**	Or equivalent single term.
		6	

Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and express equation in terms of $\tan\theta$	**M1**	
	Obtain a correct equation in $\tan\theta$ in any form	**A1**	e.g. $\frac{1-\tan^2\theta}{2\tan\theta}+\frac{1}{\tan\theta}=2$
	Reduce to $\tan^2\theta+4\tan\theta-3=0$, or 3-term equivalent	**A1**	
		3	
5(b)	Solve a 3-term quadratic for tan θ and calculate θ	**M1**	$\left(\tan\theta=-2\pm\sqrt{7}\right)$
	Obtain answer, e.g. 0.573	**A1**	Must be 3 d.p.
	Obtain second answer, e.g. 1.783 and no other	**A1**	Ignore answers outside the given interval. Treat answers in degrees as a misread. $\left(32.9°, 102.1°\right)$
		3	

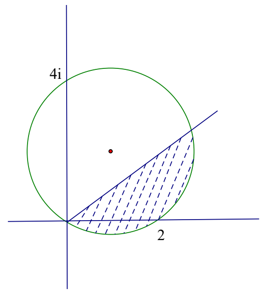
Question	Answer	Marks	Guidance
6	State or imply $1 + 2x$ as first terms of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	Form an expression for the coefficient of x or coefficient of x^2 in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	**M1**	All relevant terms considered.
	Obtain $2a + b = 3$, or equivalent	**A1**	One correct equation.
	Obtain– $2a + 2b = -6$ or equivalent	**A1**	Second correct equation.
	Obtain answer $a = 2$ and $b = -1$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given answer	**B1**	
		1	
7(b)	Correct separation of variables	**B1**	e.g. $-\int \frac{1}{t}\mathrm{d}t = \int \frac{1}{x\ln x}\mathrm{d}x$
	Obtain term $\ln(\ln x)$	**B1**	
	Obtain term $-\ln t$	**B1**	
	Evaluate a constant or use $x = \mathrm{e}$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$	**M1**	
	Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$	**A1**	
	Use log laws to enable removal of logarithms	**M1**	
	Obtain answer $x = \mathrm{e}^{\frac{2}{t}}$, or simplified equivalent	**A1**	
		7	
7(c)	State that x tends to 1 coming from $x = \mathrm{e}^{\frac{k}{t}}$	**B1**	
		1	

Question	Answer	Marks	Guidance
8(a)	Commence integration and reach $a\sqrt{x}\ln x + b\int\sqrt{x}\,.\,\frac{1}{x}\,dx$, or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x - \int 2\sqrt{x}\,.\,\frac{1}{x}\,dx$, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 6	DM1	
	Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$	A1	Obtain **given answer** from full and correct working.
		5	
8(b)	Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$	M1	e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$
	Complete the argument correctly with correct values	A1	
		2	
8(c)	Use the iterative process $a_{n+1} = \exp(\frac{1}{\sqrt{a_n}} + 2)$ correctly at least once	M1	
	Obtain answer 10.12	A1	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	A1	e.g. $10, 10.1374, 10.1156, 10.1190, \ldots.$ $9, 10.3123, 10.0886, 10.1233, 10.1178, \ldots$ $11, 9.9893, 10.1391, 10.1153, 10.1191, \ldots.$
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct method to evaluate the scalar product of relevant vectors	M1	$(-4-2+6)$
	Obtain answer zero and deduce the given statement	A1	Need a conclusion or a statement in advance that the scalar product will be zero.
		2	
9(b)	Express general point of l or m in component form, e.g. $(3 + 4s, 2 - s, 5 + 3s)$ or $(1 - t, -1 + 2t, -2 + 2t)$	B1	
	Equate at least two pairs of components and solve for s or for t	M1	
	Obtain correct answer $s = -1$ and $t = 2$	A1	
	Verify that all three equations are satisfied	A1	
	State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, or equivalent	A1	Can come from 1 correct value and no contradictory statement.
		5	

Question	Answer	Marks	Guidance
9(c)	Taking a general point P on m, form an equation in t by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\left\lvert\overrightarrow{OP}\right\rvert$ to zero, ***or*** taking a specific point Q on m, e.g. (1, – 1, – 2), using Pythagoras in triangle OPQ	***M1**	e.g. $\begin{pmatrix}1-t\\-1+2t\\-2+2t\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$
	Obtain $t=\frac{7}{9}$	**A1**	
	Carry out correct method to find OP	**DM1**	
	Obtain $\frac{\sqrt{5}}{3}$	**A1**	Obtain the **given answer** from full and correct working.
	Alternative method for question 9(c)		
	Take a specific point Q on m, e.g. (– 1, 3, 2) and use a scalar product to find QN, the projection of OQ on m	***M1**	
	Obtain $QN=\frac{11}{3}$, or equivalent	**A1**	
	Use Pythagoras to obtain ON	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	Substitute 1 + 2i in the polynomial and attempt expansions of x^2 and x^3	**M1**	$u^2 = -3 + 4i$, $u^3 = -11 - 2i$ Full substitution but need not simplify.
	Equate real and/or imaginary parts to zero	**M1**	$-18 - 3a + b = 0$, $4 + 4a = 0$
	Obtain $a = -1$	**A1**	
	Obtain $b = 15$	**A1**	
		4	
10(b)	State second root 1 – 2i	**B1**	
		1	
10(c)	State the quadratic factor $x^2 - 2x + 5$	**B1**	
	State the linear factor $2x + 3$	**B1**	
		2	
10(d)(i)	Show a circle with centre 1 + 2i	**B1**	
	Show circle passing through the origin	**B1**	
	Show the half line $y = x$ in the first quadrant (accept chord of circle)	**B1**	
	Shade the correct region on a correct diagram	**B1**	
		4	
10(d)(ii)	State answer $2 - \sqrt{5}$	**B1**	
		1	