



# Cambridge International AS & A Level

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NAME

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NUMBER

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## MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.

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- 1** Find the value of  $x$  for which  $3(2^{1-x}) = 7^x$ . Give your answer in the form  $\frac{\ln a}{\ln b}$ , where  $a$  and  $b$  are integers. [4]

- 2** Solve the inequality  $|3x - a| > 2|x + 2a|$ , where  $a$  is a positive constant. [4]

- 3 (a)** Given the complex numbers  $u = a + ib$  and  $w = c + id$ , where  $a, b, c$  and  $d$  are real, prove that  $(u + w)^* = u^* + w^*$ . [2]

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- (b) Solve the equation  $(z + 2 + i)^* + (2 + i)z = 0$ , giving your answer in the form  $x + iy$  where  $x$  and  $y$  are real. [4]

- 4 Express  $\frac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$  in partial fractions. [5]

- 5 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 3 - 2i| \leq 1$  and  $\operatorname{Im} z \geq 2$ . [4]

**(b)** Find the greatest value of  $\arg z$  for points in the shaded region, giving your answer in degrees. [3]

- 6 (a)** Using the expansions of  $\sin(3x + 2x)$  and  $\sin(3x - 2x)$ , show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$

[3]

- (b) Hence show that  $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x \, dx = \frac{1}{5}(3 - \sqrt{2})$ . [3]

- 7** The variables  $x$  and  $y$  satisfy the differential equation

$$e^{2x} \frac{dy}{dx} = 4xy^2,$$

and it is given that  $y = 1$  when  $x = 0$ .

Solve the differential equation, obtaining an expression for  $y$  in terms of  $x$ .

[7]



- 8 (a)** By first expanding  $(\cos^2 \theta + \sin^2 \theta)^2$ , show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta.$$

[3]

- (b) Hence solve the equation

$$\cos^4 \theta + \sin^4 \theta = \frac{5}{9},$$

for  $0^\circ < \theta < 180^\circ$ .

[4]

- 9** The equation of a curve is  $ye^{2x} - y^2e^x = 2$ .

(a) Show that  $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$ . [4]

- (b)** Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

- 10** With respect to the origin  $O$ , the position vectors of the points  $A$  and  $B$  are given by  $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ .

(a) Find a vector equation for the line  $l$  through  $A$  and  $B$ .

[3]

**(b)** The point  $C$  lies on  $l$  and is such that  $\overrightarrow{AC} = 3\overrightarrow{AB}$ .

Find the position vector of  $C$ .

[2]

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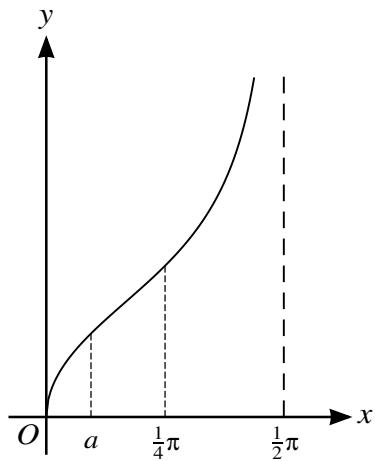
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- (c) Find the possible position vectors of the point  $P$  on  $l$  such that  $OP = \sqrt{14}$ . [5]

- 11** The equation of a curve is  $y = \sqrt{\tan x}$ , for  $0 \leq x < \frac{1}{2}\pi$ .

- (a) Express  $\frac{dy}{dx}$  in terms of  $\tan x$ , and verify that  $\frac{dy}{dx} = 1$  when  $x = \frac{1}{4}\pi$ . [4]

The value of  $\frac{dy}{dx}$  is also 1 at another point on the curve where  $x = a$ , as shown in the diagram.



- (b) Show that  $t^3 + t^2 + 3t - 1 = 0$ , where  $t = \tan a$ . [4]

(c) Use the iterative formula

$$a_{n+1} = \tan^{-1} \left( \frac{1}{3} (1 - \tan^2 a_n - \tan^3 a_n) \right)$$

to determine  $a$  correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

**Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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