Cambridge International AS & A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

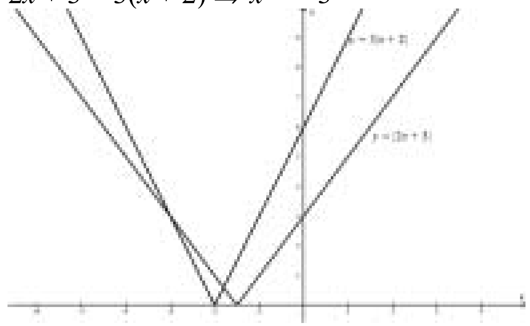
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(2x+3)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Make a reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Quadratic formula or $(5x + 9)(x + 3)$
	Obtain critical values $x = -3$ and $x = -\frac{9}{5}$	**A1**	OE
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**A1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
	Alternative method for question 1		
	Obtain critical value $x = -3$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	$2x + 3 = 3(x + 2) \Rightarrow x = -3$ 
	Obtain critical value $x = -\frac{9}{5}$ similarly	**B2**	
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**B1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
		4	

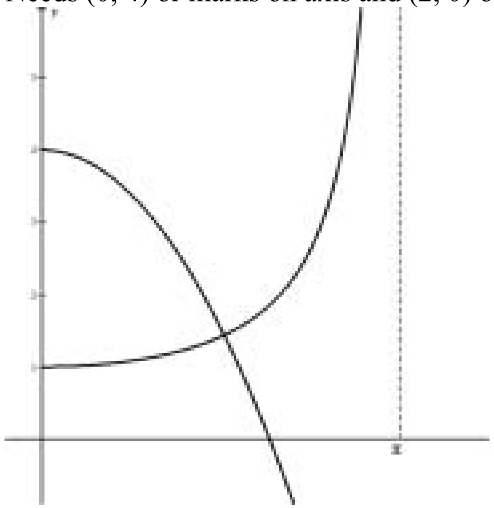
Question	Answer	Marks	Guidance
2	Show a circle with centre – 2 + 3i	**B1**	Must see (– 2, 3) or appropriate marks on axes
	Show a circle of radius 2 and centre not at the origin.	**B1**	
	Show correct **half line** from the origin	**B1**	$\frac{3\pi}{4}$ or $\frac{\pi}{4}$ seen, or half line that approximately bisects angle $\frac{\pi}{2}$.
	Shade the correct region.	**B1**	
		4	N.B. Maximum 3 out of 4 if any errors seen.

Question	Answer	Marks	Guidance
3	State or imply $n\ln x + 2\ln y = \ln C$	**B1**	
	Substitute values of ln y and ln x, or equate gradient of line to $\pm\frac{1}{2}n$, but not $\pm n$, and solve for n	**M1**	Using ln$x$ and ln$y$ values
	Obtain n = 0.8[0] or 0.8[00] or $\frac{4}{5}$	**A1**	
	Solve for C	**M1**	Using ln$x$ and ln$y$ values in equation of correct form, that is ln$C$ not C. Allow C = $e^{2.668}$.
	Obtain C = 14.41	**A1**	Must be 2 d.p.
	Alternative method for question 3		
	Obtain two correct equations in n and C by substituting x and y values in the given equation	**B1**	$(2.886)^n \times (2.484)^2 = C$ and $(1.363)^n \times (3.353)^2 = C$
	Solve for n	**M1**	Using x and y values
	Obtain n = 0.8[0] or 0.8[00] or 4/5	**A1**	$\left(\frac{2.886}{1.363}\right)^n \times \left(\frac{2.484}{3.353}\right)^2 = 1$ leading to $n = 0.7995$
	Solve for C	**M1**	Using x and y values
	Obtain C = 14.41	**A1**	Must be 2 d.p.
		5	

Question	Answer	Marks	Guidance
4	State $\frac{dx}{d\theta} = \sin\theta$ or $\frac{dy}{d\theta} = -\sin\theta + \frac{1}{2}\sin 2\theta$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain correct answer in any form	**A1**	e.g. $\dfrac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta}$
	Use double angle correctly to obtain $\frac{dy}{dx}$ in terms of θ	**M1**	$\sin 2\theta = 2\sin\theta\cos\theta$
	Obtain the given answer with no errors seen $-2\sin^2\left(\frac{1}{2}\theta\right)$	**A1**	AG. Requires correct cancellation of ALL $\sin\theta$ terms and $\cos\theta = 1 - 2\sin^2\left(\frac{1}{2}\theta\right)$ seen **SC** For incorrect signs, consistent throughout max. B0, M1, A0, M1, A1
		5	

Question	Answer	Marks	Guidance
5	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\alpha$ and $\tan\beta$	**M1**	$\frac{\tan + \tan}{1 \ \tan\tan} = 2$
	Substitute throughout for $\tan\alpha$ or for $\tan\beta$	**M1**	$\frac{3\tan + \tan}{1 \ 3\tan^2} = 2$
	Obtain $3\tan^2\beta + 2\tan\beta - 1 = 0$ or $\tan^2\alpha + 2\tan\alpha - 3 = 0$	**A1**	OE e.g. $6\tan^2\beta + 4\tan\beta - 2 = 0$ or $\frac{2}{3}\tan^2\alpha + \frac{4}{3}\tan\alpha - 2 = 0$
	Solve a 3-term quadratic and find an angle	**M1**	
	Obtain answer $\alpha = 45°$, $\beta = 18.4°$	**A1**	$\frac{\pi}{4}$ or 0.785, 0.322
	Obtain answer $\alpha = 108.4°$, $\beta = 135°$	**A1**	1.89, $\frac{3\pi}{4}$ or 2.36. Answer in radians, max. A1A0 or vice versa. Ignore answers outside [0°, 180°]
		6	SC: If A0A0 allow **SC B1** for both α's or both β's

Question	Answer	Marks	Guidance
6	Substitute and obtain a correct equation in x and y	**B1**	$(x + \mathrm{i}y)^2 + 2\mathrm{i}(x - \mathrm{i}y) = 1$
	Use $\mathrm{i}^2 = -1$ at least once and equate real and imaginary parts	**M1**	
	Obtain two correct equations, e.g. $x^2 - y^2 + 2y = 1$ and $2xy + 2x = 0$	**A1**	
	Solve for x or for y	**M1**	
	Using $y = -1$, obtain answer $w = -2 - \mathrm{i}$ only	**A1**	A0 if $w = 2 - \mathrm{i}$ as well
	Using $x = 0$, obtain answer $w = \mathrm{i}$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Sketch a relevant graph, e.g. $y = 4 - x^2$	**B1**	Needs (0, 4) or marks on axis and (2, 0) or (π, 0) 
	Sketch a second relevant graph, e.g. $y = \sec\frac{1}{2}x$, and justify the given statement	**B1**	Needs (0, 1) or mark on axis and (π, 0) Asymptote NOT required, but must NOT reach $x = \pi$. Sec graph must exist over at least interval $\left[0, \frac{3\pi}{4}\right]$ and quadratic graph over [0, 2.5].
		2	
7(b)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 1$ and $x = 2$.	**M1**	Need all 4 values or the 2 values correct for M1. Angles in degrees score M0.
	Complete the argument with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative process correctly at least twice	**M1**	
	Obtain final answer 1.60	**A1**	Must be 2 d.p.
	Show sufficient iterations to 4 d.p.to justify 1.60 to 2 d.p. or show there is a sign change in the interval (1.595, 1.605)	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	Commence division and reach quotient of the form $2x \pm 1$	**M1**	Or by inspection $8x^3 + 4x^2 + 2x + 7 = (4x^2 + 1)(2x \pm 1) + r$
	Obtain (quotient) $2x + 1$	**A1**	
	Obtain (remainder) 6	**A1**	
		3	

Question	Answer	Marks	Guidance
8(b)	Obtain terms $x^2 + x$	**B1**	OE
	Obtain term of the form $a\tan^{-1}2x$	**M1**	
	Obtain term $3\tan^{-1}2x$	**A1**	OE
	Use $x = 0$ and $x = \frac{1}{2}$ as limits in a solution containing a term of the form $a\tan^{-1}2x$	**M1**	$\left(\frac{1}{2}\right)^2 + \frac{1}{2} + a\frac{\pi}{4}$, need $\frac{\pi}{4}$ seen or implied
	Obtain final answer $\frac{3}{4}(1+\pi)$, or exact equivalent	**A1**	ISW, Answers in degrees score A0.
		5	

Question	Answer	Marks	Guidance
9	Correctly separate variables and integrate at least one side	**M1**	To obtain $a\ln y$ or $b\ln(x+1) + c\ln(3x+1)$
	Obtain term $\ln y$ from integral of $1/y$	**B1**	
	State or imply the form $\dfrac{A}{x+1} + \dfrac{B}{3x+1}$ and use a correct method to find a constant	**M1**	
	Obtain $A = -\dfrac{1}{2}$ and $B = \dfrac{3}{2}$	**A1**	
	Obtain terms $-\dfrac{1}{2}\ln(x+1)+\dfrac{1}{2}\ln(3x+1)$ or $-\dfrac{1}{2}\ln(2x+2)+\dfrac{1}{2}\ln(6x+2)$ or combination of these terms	**A1 FT + A1 FT**	The FT is on the values of A and B.
	Use $x = 1$ and $y = 1$ to evaluate a constant, or expression for a constant, (decimal equivalent of ln terms allowed) or as limits, in a solution containing terms $a\ln y$, $b\ln(x+1)$ and $c\ln(3x+1)$, where $abc \neq 0$	***M1**	e.g. $\ln y = -\dfrac{1}{2}\ln(x+1)+\dfrac{1}{2}\ln(3x+1)-\dfrac{1}{2}\ln 2$
	Obtain an expression for y or y^2 and substitute $x = 3$	**DM1**	Do not accept decimal equivalent of ln terms
	Obtain answer $y = \dfrac{1}{2}\sqrt{5}$ or $\sqrt{\dfrac{5}{4}}$ or $\sqrt{\dfrac{10}{8}}$	**A1**	ISW. Must be simplified and exact, do not allow 1.118 or $e^{\frac{1}{2}\ln\frac{5}{4}}$.
		9	

Question	Answer	Marks	Guidance
10(a)	Obtain direction vector $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$	**B1**	OE
	Use a correct method to form a vector equation	**M1**	
	Obtain answer $\mathbf{r}=2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$ or $\mathbf{r}=\ \mathbf{i}\ \ 2\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$	**A1**	Need **r** or r on LHS
		3	
10(b)	Carry out the correct process for evaluating the scalar product of the direction vectors.	**M1**	(–1, – 3, 1) **.** (1, –3, –2) = – 1 + 9 – 2
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result for any 2 vectors	**M1**	$\cos^{-1}\left(\frac{1+9\ \ 2}{(1+9+1)(1+9+4))}\right)$
	Obtain answer 61.1 °	**A1**	61.086°
		3	

Question	Answer	Marks	Guidance
10(c)	Express general point of AB or l in component form, e.g. $(2-\lambda, 1-3\lambda, 1+\lambda)$ or $(1+\mu, 2-3\mu, -3-2\mu)$	B1	
	Equate at least two pairs of components and solve for λ or for μ	M1	
	Obtain a correct answer for λ or μ, e.g. $\lambda = 6, \frac{1}{3}$, or $-\frac{14}{9}$; $\mu = -5, \frac{2}{3}$ or $-\frac{11}{9}$	A1	
	Verify that all three equations are not satisfied, and the lines do not intersect	A1	
	Express general point of AB or l in component form, e.g. $(1-\lambda^*, -2-3\lambda^*, 2+\lambda^*)$ or $(1+\mu^*, 2-3\mu^*, -3-2\mu^*)$	4	

Question	Answer	Marks	Guidance
11(a)	Use correct product rule or chain rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\cos x.\cos 2x - \sin x.2\sin 2x$
	Equate derivative to zero and use a correct double angle formula	***M1**	If chain rule used then derivative set to 0 gains M1 since correct double angle formula has already been used.
	Obtain an equation in one trigonometric variable	**DM1**	Allow following from coefficient errors in differentiation only
	Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**	One of these 3 expressions
	Obtain final answer $x = 0.421$	**A1**	Must be 3s.f.
		6	

Question	Answer	Marks	Guidance
11(b)	State or imply du = $-\sin x$ dx	**B1**	
	Using double angle formula, express integral in terms of u and du	**M1**	Use $\cos 2x = 2\cos^2 x - 1$
	Integrate and obtain $\pm\left(u - \frac{2}{3}u^3\right)$	**A1**	
	Use limits $u = 1$, $u = \frac{1}{\sqrt{2}}$ in an integral of the form $au + bu^3$, where $ab \neq 0$	**M1**	Require both limits substituted twice in $au + bu^3$ for M1. Do not condone decimals.
	Obtain $\frac{1}{3}\left(\sqrt{2} - 1\right)$ or $\frac{1}{3}\sqrt{2}\ \ \frac{1}{3}$ or $\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)\frac{1}{3}$ or simplified equivalent	**A1**	ISW
		5	