Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

CANDIDATE
NUMBER

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

1 Solve the equation $2(3^{2x-1}) = 4^{x+1}$, giving your answer correct to 2 decimal places. [4]

2 **(a)** Expand $(2 - x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) State the set of values of x for which the expansion is valid. [1]

....

....

....

....

....

3 Solve the equation $2 \cot 2x + 3 \cot x = 5$, for $0° < x < 180°$. [6]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

4 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for y in terms of x. [7]

9709/31/M/J/22

5 The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by p(x). It is given that $(x - 2)$ is a factor of both p(x) and p$'(x)$.

(a) Find the values of a and b. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) When a and b have these values, factorise p(x) completely. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

6 Let $I = \displaystyle\int_0^3 \dfrac{27}{(9+x^2)^2}\,dx$.

(a) Using the substitution $x = 3\tan\theta$, show that $I = \displaystyle\int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Hence find the exact value of I. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

7 The complex number u is defined by $u = \dfrac{\sqrt{2} - a\sqrt{2}\,i}{1 + 2i}$, where a is a positive integer.

(a) Express u in terms of a, in the form $x + iy$, where x and y are real and exact. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

It is now given that $a = 3$.

(b) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [2]

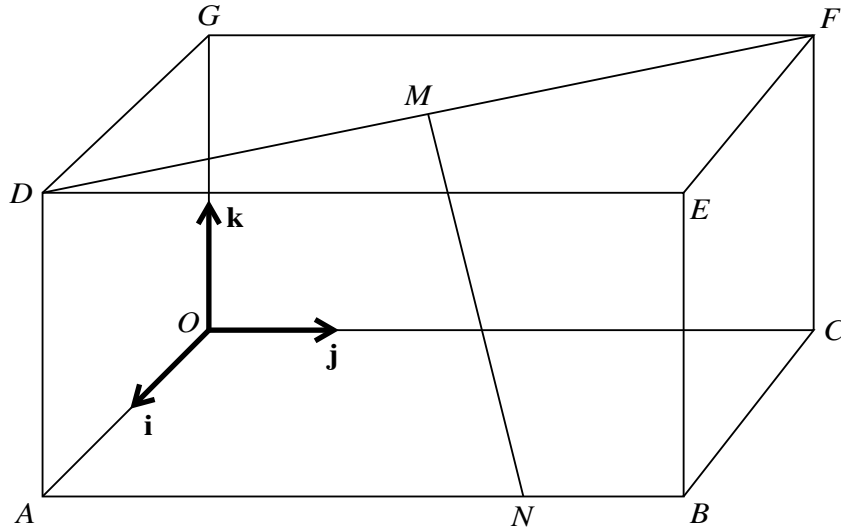..

....

....

....

....

....

....

....

....

....

....

(c) Using your answer to part **(b)**, find the two square roots of u. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

....

....

....

....

....

....

....

....

....

....

8 The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\dfrac{dy}{dx}$ in terms of x and y. [4]

The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle α.

(b) Find the exact value of $\tan \alpha$. [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

9

In the diagram, *OABCDEFG* is a cuboid in which *OA* = 2 units, *OC* = 4 units and *OG* = 2 units. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *OG* respectively. The point *M* is the midpoint of *DF*. The point *N* on *AB* is such that *AN* = 3*NB*.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

....

....

....

....

....

....

....

(b) Find a vector equation for the line through *M* and *N*. [2]

....

....

....

....

....

....

(c) Show that the length of the perpendicular from O to the line through M and N is $\sqrt{\frac{53}{6}}$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

10
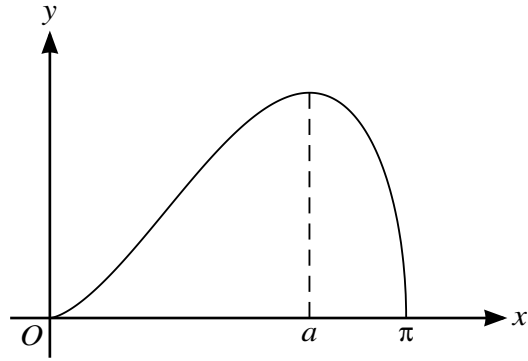
The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

(a) Show that $\tan a = -\frac{1}{2}a$. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Verify by calculation that a lies between 2 and 2.5. [2]

(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}\left(\frac{1}{2}x_n\right)$ converges, then it converges to a, the root of the equation in part **(a)**. [2]

(d) Use the iterative formula given in part **(c)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....