Cambridge Assessment International Education

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

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(b) Find the equation of the circle with centre A which passes through B. [3]

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2 The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

3 **(a)** Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

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(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

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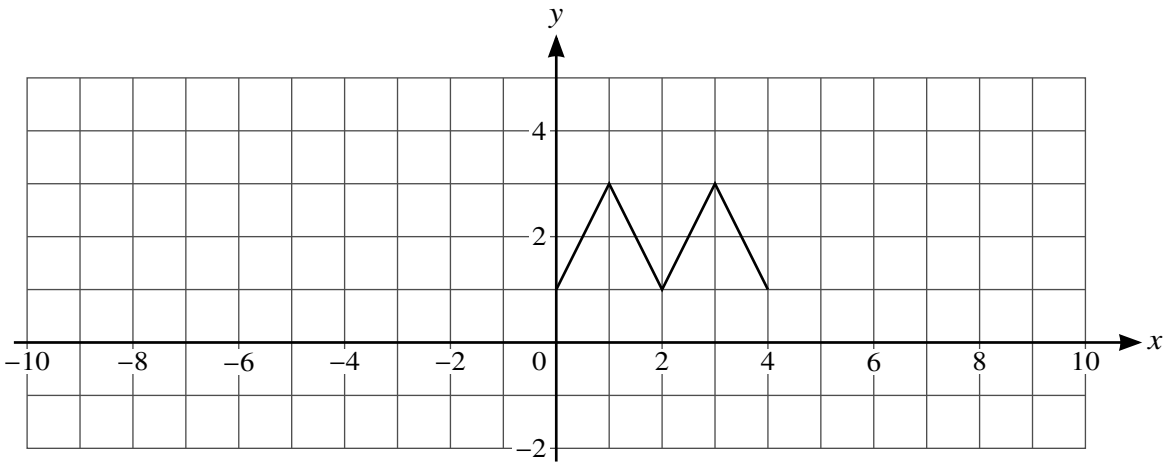
4 A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

5 The graph with equation $y = f(x)$ is transformed to the graph with equation $y = g(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = f(x)$.

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

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6 The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

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(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

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(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

7 **(a)** Prove the identity $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$. [3]

(b) Hence find the exact solutions of the equation $\frac{\sin\theta}{\sin\theta+\cos\theta}+\frac{\cos\theta}{\sin\theta-\cos\theta}=2$ for $0\leqslant\theta\leqslant\pi$. [4]

8 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

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(c) State the set of values of x for which y increases as x increases. [1]

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9 Functions f and g are defined by

$$\mathrm{f}(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$\mathrm{g}(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $\mathrm{gf}(x)$. [1]

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(b) Given that $\mathrm{gf}(2) = 11$, find the value of a. [2]

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(c) Given that the graph of $y = \mathrm{f}(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

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(e) Explain why the composite function fg cannot be formed. [1]

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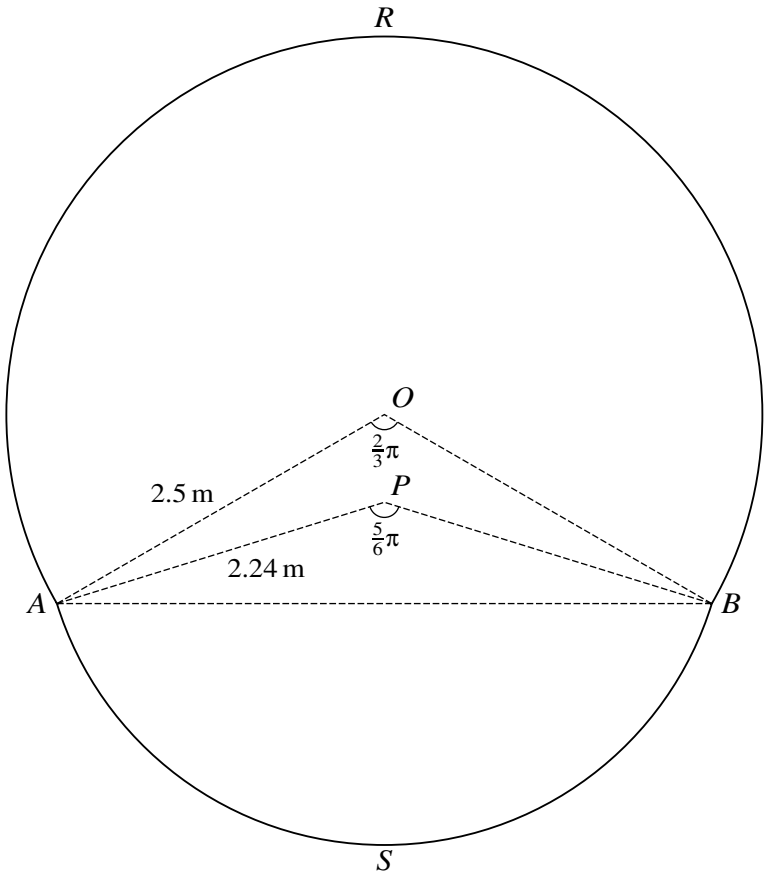

The diagram shows a cross-section $RASB$ of the body of an aircraft. The cross-section consists of a sector $OARB$ of a circle of radius 2.5 m, with centre O, a sector $PASB$ of another circle of radius 2.24 m with centre P and a quadrilateral $OAPB$. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section $RASB$, giving your answer correct to 2 decimal places. [3]

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(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

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(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

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11 **(a)** Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]

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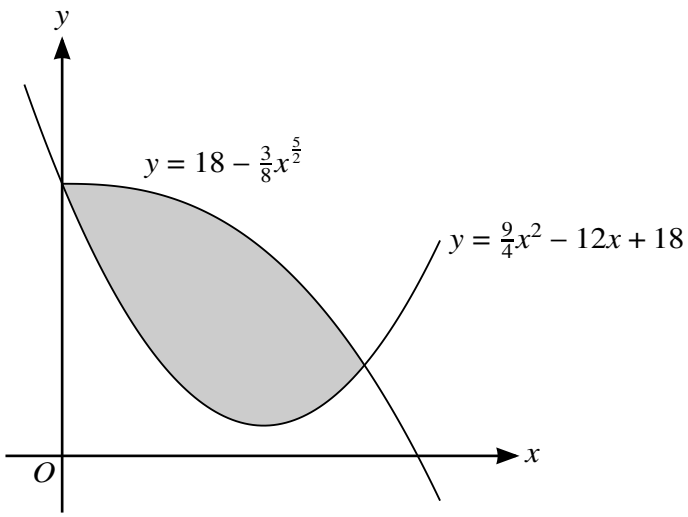

The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

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(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

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Additional Page

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