

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

MATHEMATICS 9709/32

Paper 3 Pure Mathematics 3

February/March 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages.

It	is given that $x = \ln(2y - 3) - \ln(y + 4)$.
E	xpress y in terms of x .
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2

(a) On an Argand diagram, shade the region whose points represent complex numbers z satisfying

	the inequalities $-\frac{1}{3}\pi \le \arg(z-1-2i) \le \frac{1}{3}\pi$ and $\operatorname{Re} z \le 3$.	[3]
(b)	Colculate the least value of are a fer points in the region from (a). Give your encycer in rec	liona
(b)	Calculate the least value of $\arg z$ for points in the region from (a). Give your answer in rac correct to 3 decimal places.	dians [2]
(b)	Calculate the least value of $\arg z$ for points in the region from (a). Give your answer in rac correct to 3 decimal places.	
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3

The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by p(x). When p(x) is

Find the values of a and b .	[5

4	Solve	the	ea	uation

$\frac{5z}{1+2i} - zz^* + 30 + 10i = 0,$	
giving your answers in the form $x + iy$, where x and y are real.	[5]
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5	The	parametric	equations	of a	curve	are

2t	2 -
$x = te^{2t}$,	$y = t^2 + t + 3$

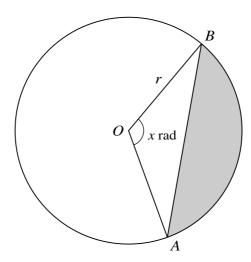
(a)	Show that $\frac{dy}{dx} = e^{-2t}$.	[3]
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(b)	Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0, 3 - \frac{1}{e^4}\right)$.

6 (a)	Express $5 \sin \theta + 12 \cos \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.	[3]
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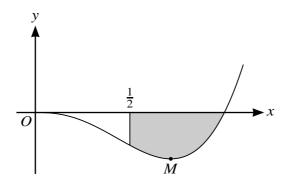
7



The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a)	Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$.	[4]

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The diagram shows the curve $y = x^3 \ln x$, for x > 0, and its minimum point M.

(a)	Find the exact coordinates of M .	[4]
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(b)	Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = \frac{1}{2}$. [5]

9 The variables x and y satisfy the differential equation
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$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{3y}\sin^2 2x.$$

It is given that y = 0 when x = 0.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$.	[7]

10	With respect to th	e origin Q , the	points A. B. C	and D have	position vectors	given by
10	William respect to th	ic origin o, me	pomis 11, D , C	and D mave	position vectors	given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \qquad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

a)	Find the obtuse angle between the vectors \overrightarrow{OA} and \overrightarrow{OB} .
he	line l passes through the points A and B .
b)	Find a vector equation for the line l .

C and D .						
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11	Let $f(r) = \frac{5}{2}$	$x^2 + x + 11$
11	Let $f(x) = \frac{5}{4}$	$+x^2)(1+x)$

(a)	Express $f(x)$ in partial fractions.	[5]
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Hence show that $\int_0^2 f(x) dx = \ln 54 - \frac{1}{8}\pi.$	
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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.		

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