Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

JC23 06_9709_21/RP

1 Use logarithms to solve the equation $12^x = 3^{2x+1}$. Give your answer correct to 3 significant figures. [4]

2 A curve has equation $y = \dfrac{2 + 3\ln x}{1 + 2x}$.

Find the equation of the tangent to the curve at the point $\left(1, \frac{2}{3}\right)$. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [5]

3 It is given that $\int_0^a (3e^{2x} - 1)\,dx = 12$, where a is a positive constant.

(a) Show that $a = \frac{1}{2}\ln\left(9 + \frac{2}{3}a\right)$. [4]

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(b) Use an iterative formula, based on the equation in **(a)**, to find the value of a correct to 4 significant figures. Use an initial value of 1 and give the result of each iteration to 6 significant figures. [3]

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4 The polynomial p(x) is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that $(x - 3)$ is a factor of p(x).

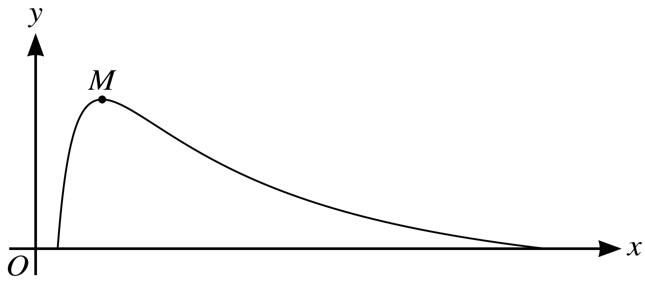
(a) Find the value of k. [2]

(b) Hence find the quotient when p(x) is divided by $(x - 3)$ and factorise p(x) completely. [3]

(c) It is given that a is one of the roots of the equation $p(x) = 0$.

Given also that the equation $|4y - 5| = a$ is satisfied by two real values of y, find these two values of y. [3]

5



The diagram shows the curve with parametric equations

$$x = 4e^{2t}, \qquad y = 5e^{-t}\cos 2t,$$

for $-\frac{1}{4}\pi \leqslant t \leqslant \frac{1}{4}\pi$. The curve has a maximum point M.

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. [3]

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(b) Find the coordinates of M, giving each coordinate correct to 3 significant figures. [5]

6 Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \left(4\cos^2 2x + \frac{1}{\cos^2 x}\right) \mathrm{d}x = \frac{3}{4}\sqrt{3} + \frac{1}{6}\pi - 1.$ [7]

7 **(a)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(b) Solve the equation $7\cos\theta + 24\sin\theta = 18$ for $0° < \theta < 360°$. [4]

(c) As β varies, the greatest possible value of

$$\frac{150}{7\cos\frac{1}{2}\beta + 24\sin\frac{1}{2}\beta + 50}$$

is denoted by V.

Find the value of V and determine the smallest positive value of β (in degrees) for which the value of V occurs. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.