Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1+18x+135x^2$	**B2, 1, 0**	Accept $1,\ 18x,\ 135x^2$ listed horizontally or vertically or $1x^0+18x+135x^2$.
		2	
1(b)	Coefficient of x^2 is $135-7\times18+1=10$	**M1 A1**	3 products, allow $10x^2$. If full expansion given, like terms must be collected for M1.
		2	

Question	Answer	Marks	Guidance
2	$cx^2+3x-c=2cx+3$ leading to $cx^2+(3-2c)x-(c+3)\ \ [=0]$	**M1**	Forming a 3-term quadratic, all terms on one side.
	$b^2-4ac=(3-2c)^2+4c(c+3)$	**M1**	2nd M1 for b^2-4ac correct for *their* <u>a</u>, b, c i.e. no sign errors.
	$= 8c^2+9$	**A1**	
	> 0 [for all values of c] leading to B [Intersects for all values of c]	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
3	$\frac{dV}{dx} = 3x^2$	**B1**	SOI
	$\frac{dV}{dt}\left[=\frac{dV}{dx}\times\frac{dx}{dt}\right] = 3\times 20^2\times 0.01$	**M1**	Correct use of chain rule with $x = 20$ substituted into $\frac{dV}{dx}$.
	12	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$\{-(x-3)^2\}\ \{-1\}$	**B1 B1**	OE. Must be a quadratic e.g. $3x - 1$ B0 B0. **SC B1** for correct use of generalised function notation.
		2	
4(b)	$\{-(x-3)^2\}\ \{+1\}$	**B1 B1**	OE. Must be a quadratic. **SC B1** for correct use of generalised function notation.
		2	
4(c)	{Translation} $\begin{pmatrix}\{0\}\\\{2\}\end{pmatrix}$	**B2, 1, 0**	FT from **(a)** and **(b)** if a translation parallel to the y axis. B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		2	

Question	Answer	Marks	Guidance
5(a)	$4\sin^2 x + 5\cos x + 2 \quad [=0]$	***M1**	Multiply by $\sin x$ (or writing as a single fraction) and using $\tan x = \dfrac{\sin x}{\cos x}$.
	$4(1-\cos^2 x) + 5\cos x + 2 \quad [=0]$	**DM1**	Correctly obtaining a quadratic in $\cos x$ (allow sign errors).
	$4\cos^2 x - 5\cos x - 6 = 0$	**A1**	Condone missing x. Must be = 0 unless 0 appears on RHS earlier.
		3	
5(b)	$(4\cos x + 3)(\cos x - 2) \quad [=0]$	**M1**	Or use of formula or completing square.
	138.6°, 221.4°	**A1 B1 FT**	FT on 360° – 1st solution from quadratic in $\cos x$. Use of radians (2.42) A0 but allow B1 FT for 2π: 1st solution if use of radians is clear. **SC** If M0 scored **SC B1 B1** for correct final answer(s). If extra incorrect solutions in the range 0 → 360° are given award A1 B0.
		3	

Question	Answer	Marks	Guidance
6(a)	$k = \frac{2}{3}$	**B1**	Allow ACB = $\frac{2\pi}{3}$.
		1	
6(b)	Perimeter of shaded area $= 2\pi r$	**B1**	
		1	

Question	Answer	Marks	Guidance
6(c)	Major sector $OAB = \frac{1}{2}r^2 \times \frac{4\pi}{3}$	*M1	Expect $\frac{2}{3}\pi r^2$. Finds area of any relevant sector or triangle. Can be embedded in segment formula.
	One or both segments $= [2] \times \left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin\frac{\pi}{3}\right)$	*M1	
	$= [2]\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)$	A1	
	Shaded area $= \frac{2}{3}\pi r^2 - 2\left(\frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4}\right)$	DM1	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	A1	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Sector $CAOB = [2] \times \frac{1}{2} r^2 \textit{their} \frac{1}{3}\pi$	***M1**	Expect [2] × $\frac{1}{6}\pi r^2$. Can be embedded in segment formula.
	One or both segments $= [2] \times \left(\frac{1}{2} r^2 \times \frac{\pi}{3} - \frac{1}{2} r^2 \sin\frac{\pi}{3} \right)$	***M1**	
	$= [2]\left(r^2 \frac{\pi}{6} - r^2 \frac{\sqrt{3}}{4} \right)$	**A1**	
	Shaded area = $\pi r^2 - \left\{ \frac{1}{3}\pi r^2 + 2\left(r^2 \frac{\pi}{6} - r^2 \frac{\sqrt{3}}{4} \right) \right\}$	**DM1**	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	**A1**	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Area of rhombus AOBC $=[2]\times\frac{1}{2}r^2\sin\frac{\pi}{3}$	**M1**	Expect [2] $\times$ $\frac{\sqrt{3}}{4}$. Can be embedded in segment formula.
	One or both segments $=[2]\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\sin\frac{\pi}{3}\right)$	**M1**	
	$=[2]\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area = $\pi r^2-\left\{\frac{\sqrt{3}}{2}r^2-4\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$=\frac{\pi r^2}{3}+\frac{r^2\sqrt{3}}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
7	$a(1+r)=15$	**B1**	Accept $\frac{a(1-r^2)}{1-r}=15$ for first B1.
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{125}{7}(1-r)(1+r)=15$	**M1**	Eliminate *a*.
	$1-r^2=\frac{105}{125}$	**M1**	
	$r^2=\frac{4}{25}$ leading to $r=-\frac{2}{5}$	**A1**	Condone $\frac{2}{5}$ or $\pm\frac{2}{5}$.
	$a=\frac{125}{7}\times\frac{7}{5}=25$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$7(15-15r) = (125 - 125r)(1 - r^2)$	**M1**	
	$125r^3 - 125r^2 - 20r + 20 = 0$	**M1**	
	$r = \frac{-2}{5}$ $[1, \frac{2}{5}]$	**A1**	Condone extra 'answer' of r = 1.
	a = 25	**A1**	Ignore 2nd answer.
	3rd term = 4	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{a}{1-\left(\frac{15}{a}-1\right)}=\frac{125}{7}$	**M1**	Eliminate *r*.
	$7a^2 - 250a + 1875$ [= 0]	**M1**	
	a = 25, $\left[\frac{75}{7}\right]$	**A1**	Condone extra 'answer' of $r = \left[\frac{75}{7}\right]$.
	r = $\frac{-2}{5}$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO
		7	

Question	Answer	Marks	Guidance
8(a)	$u = 2x - 3$ leading to $2u^4 = u^2 + 1$ leading to $2u^4 - u^2 - 1 [= 0]$	**B1**	
	$(2u^2 + 1)(u^2 - 1)[= 0]$	**M1**	Factors or formula or completing square must be shown.
	$u = \pm 1$ leading to $2x - 3 = \pm 1$ leading to $x = 1$ or 2	**A1**	
	(1, 2), (2, 2)	**A1**	**Special case:** If B1 M0 scored then **SC B2** can be awarded for correct coordinates or **SC B1** for correct x values only.
			Special case $2(2x - 3)^4 = (2x - 3)^2 + 1$ $32x^4 - 192x^3 + 428x^2 - 420x + 152 = 0$ $x = 1, 2$ finding both from a correct quartic **SC B1** (1, 2), (2, 2) **SC DB1** **Special case:** Trial and improvement without quartic. Both x values correct B1, both coordinates correct B2.
		4	

Question	Answer	Marks	Guidance
8(b)	$\left\{\dfrac{(2x-3)^3}{3\times 2}+x\right\}[-]\left\{\dfrac{2(2x-3)^5}{5\times 2}\right\}$	**B1 B1**	Integrate the 2 functions.
	$\left(\dfrac{1}{6}+2\right)-\left(-\dfrac{1}{6}+1\right)-\left\{\dfrac{1}{5}-\left(-\dfrac{1}{5}\right)\right\}$	**M1**	Apply *their* limits 1 → 2 (must be shown) to an integral. Some evidence of substitution. Minimum $(\frac{13}{6}-\frac{5}{6})-(\frac{1}{5}+\frac{1}{5})$ or equivalent. Allow 1 sign error for 1st M1.
	$\dfrac{4}{3}-\dfrac{2}{5}$	**M1**	Subtract (at some point) the 2 areas. Must subtract **areas** and not just integrals.
	$\dfrac{14}{15}$	**A1**	**Special case:** If M0 for substitution of limits can award **SC B1** for correct answer. Condone $-\frac{14}{15}$ if corrected.
			If subtraction is the wrong way round award B1 B1 M1 M1 A0. $\int y^2\,dx$ or $\int x\,dy$ scores 0 /5. $\pi\int y\,dx$ used. Award B1 B1 M1 M1 A0.

Question	Answer	Marks	Guidance
8(b)	**Alternative method for Question 8(b)**		
	u = 2x – 3 $\int\left(u^2+1-2u^4\right)\mathrm{d}u$ $\left\{\frac{1}{2}\right\}\left(\left\{\frac{1}{3}u^3+u\right\}-\left\{\frac{2}{5}u^5\right\}\right)$	**B2,1,0**	
	$\frac{1}{2}\left(\left(\frac{1}{3}+1-\frac{2}{5}\right)-\left(\frac{-1}{3}-1+\frac{2}{5}\right)\right)$	**M1**	Applies limits –1 → 1.
		M1	Subtract (at some point) the 2 areas.
	$\frac{1}{2}\left(\frac{14}{15}+\frac{14}{15}\right)$ $\frac{14}{15}$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	$(2x-3)^2+4$	**B1 B1**	Or a = –3, b = 4.
		2	

Question	Answer	Marks	Guidance
9(b)	*their* $(2x-3)^2+4[<]8$ OR $4x^2-12x+13[<]8$	*M1	Linking quadratic with 8.
	$(2x-3)^2<4$ leading to $-2<2x-3<2$ OR $4x^2-12x+5<0$ leading to $(2x-1)(2x-5)<0$	DM1	Simplify to 3-term quadratic and solve. Condone no method shown.
	$\frac{1}{2}<x<2\frac{1}{2}$ leading to [LEAST] $p=\frac{1}{2}$, [GREATEST] $q=2\frac{1}{2}$	A1	
		3	
9(c)	$\text{gf}(x)=12x^2-36x+40$	B1	OE $\text{gf}(x)=3(2x-3)^2+13$.
		1	
9(d)	$y=(2x-3)^2+4$ leading to $(2x-3)^2=y-4$ leading to $2x-3=[\pm]\sqrt{y-4}$	*M1	
	$2x=3[\pm]\sqrt{y-4}$ leading to $x=\frac{3}{2}[\pm]\frac{\sqrt{y-4}}{2}$	DM1	
	$\text{h}^{-1}(x)=\frac{3}{2}-\frac{\sqrt{x-4}}{2}$	A1	
		3	

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = 3x^2 \ [+c]$	B1	
	$3 \times 2^2 + c = 0$	M1	Substitute $x = 2$ and $\frac{dy}{dx} = 0$ into an integral (*c* must be present).
	$\frac{dy}{dx} = 3x^2 \ -12$	A1	
		3	
10(b)	$y = x^3 - 12x \ [+k]$	B1 FT	FT on *their* non-zero *c* (dependent on *c* being found at some stage).
	$-10 = 2^3 - 12 \times 2 + k$	M1	Substitute $x = 2$, $y = -10$ (*k* present).
	$y = x^3 - 12x + 6$	A1	Must be *y* = (unless $y = x^3 - 12x + k$ stated earlier).
		3	

Question	Answer	Marks	Guidance
10(c)	$3x^2 - 12 = 0$ [leading to $x = -2$]	M1	Set *their* two term $\frac{dy}{dx} = 0$. Expect $x = -2$. Ignore x = 2 given in addition.
	$y = (-2)^3 - 12 \times (-2) + 6 = 22$ leading to (–2, 22)	A1	
	When $x = -2$, $\frac{d^2y}{dx^2} < 0$ (or –12) hence Maximum	A1	Can be from correct conclusion from $\frac{dy}{dx}$ **sign** diagram if $\frac{dy}{dx}$ calculated correctly. Do not allow concave downward for final A1. Can be awarded if the only error is incorrect or missing y-coordinate.
		3	
10(d)	At $x = 0$, $\frac{dy}{dx} = -12$, $y = 6$	M1	Both required. FT on *their* $\frac{dy}{dx}$ and y.
	$y - 6 = -12x$	A1	OE
		2	

Question	Answer	Marks	Guidance
11(a)	Gradient of *AB* = –1	B1	SOI
	Centre of circle = (4, –1)	B1	SOI
	Equation of *AB* is $y + 1 = -1(x - 4)$ leading to $y = -x + 3$	B1 FT	FT *their* centre with gradient –1.
		3	

Question	Answer	Marks	Guidance
11(b)	$(x-4)^2+(-x+3+1)^2=40$	*M1	Substitute *their AB* into circle equation.
	$2(x-4)^2=40$ OR [2] (x^2-8x-4) leading to $\frac{8\pm\sqrt{64+16}}{2}$ or $\frac{16\pm\sqrt{256+64}}{4}$	DM1	Forming and solving 3-term quadratic.
	$x=4[\pm]\sqrt{20}$	A1	OE. No fractions.
	$(4-\sqrt{20},-1+\sqrt{20})$	A1	OE **Special case:** If M1 M0 scored then **SCB2** can be awarded for correct coordinates or **SCB1** for correct *x* values only. Ignore other coordinate
		4	
11(c)	$y-\textit{their}(-1+\sqrt{20})=1\{x-\textit{their}(4-\sqrt{20})\}$	M1	OE
	$y=x-5+2\sqrt{20}$ or $y=x-5+\sqrt{80}$ or $y=x-5+4\sqrt{5}$	A1	
		2	