Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **23** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	[Coefficient of x^3 from $(3+2ax)^5$ =] $10\times9\times8a^3$ $\left[=720a^3\right]$	**B1**	May be seen in an expansion or with x^3.
	[Coefficient of x^2 from $(2+ax)^6$ =] $15\times16\times a^2$ $\left[=240a^2\right]$	**B1**	May be seen in an expansion or with x^2.
	$\mathit{their}\left(10\times9\times8a^3\right)=6\times \mathit{their}\left(15\times16\times a^2\right)$ $\left[\Rightarrow 720a^3=1440a^2\right]$	**M1**	OE Equating *their* coefficient of x^3 and 6 × *their* coefficient of x^2.
	$\boldsymbol{a}$ **= 2**	**A1**	Condone extra solution $a = 0$.
		4	

Question	Answer	Marks	Guidance
2	[$\tan^{-1}4x=$]$\left(\mathit{their}-\dfrac{\pi}{6}\right)\pm\dfrac{\pi}{6}$ $\left[\tan^{-1}4x=\pm\dfrac{\pi}{3},\pm1.047\text{ or }0\right]$	**M1**	OE Evaluating $\left(-\cos^{-1}\dfrac{\sqrt{3}}{2}\right)$ in rad and adding or subtracting $\dfrac{\pi}{6}$. Allow working with **both** angles in degrees.
	$\left[4x=-\sqrt{3},\ x=\right]-\dfrac{\sqrt{3}}{4}$	**A1**	**Note:** answer of $-0.43\text{ or }\dfrac{\sqrt{3}}{4}$ implies M1
		2	

Question	Answer	Marks	Guidance
3(a)	[Gradient of normal =] $\dfrac{-1}{\text{Their}\,\frac{11}{2}}$ $\left[\dfrac{-1}{\frac{11}{2}}=-\dfrac{2}{11}\right]$	**M1**	Tangent gradient must come from $x = 2$ substituted into the given expression.
	$\dfrac{y-8}{x-2}=-\dfrac{2}{11}$ or $11y+2x=92$ or $y=-\dfrac{2x}{11}+\dfrac{92}{11}$	**A1**	OE
		2	
3(b)	$[y=]\left\{\dfrac{1}{2}x^2\div 2\right\}\left\{+\dfrac{72}{x^3}\div -3\right\}$ $[+c]$ $\left[\dfrac{x^2}{4}-\dfrac{24}{x^3}+c\right]$	**B1, B1**	One mark for each correct unsimplified { }.
	$8=\dfrac{1}{4}\times 4-\dfrac{24}{8}+c$	**M1**	Substitution of $x = 2$, $y = 8$ into *their* integrated expression, defined by at least one correct power. Two terms and + c needed.
	$y=\left(\dfrac{1}{4}\text{ or }0.25\right)x^2-\dfrac{24}{x^3}+10$	**A1**	Both coefficients must be simplified but allow x^{-3}. Condone $c = 10$ as line as long as either y or f(x) = is seen elsewhere.
		4	

Question	Answer	Marks	Guidance
4(a)	[Arc length =] $2\times\dfrac{\pi}{3}$ or $\dfrac{60}{360}\times 2\pi\times 2$	**B1**	Finding one correct arc length – may be implied by correct final answer.
	[Perimeter =] 2π or 6.28	**B1**	AWRT
		2	

Question	Answer	Marks	Guidance
4(b)	[Area of one sector =] $\frac{1}{2}\times 2^2\times\frac{\pi}{3}$ or $\frac{60}{360}\times\pi\times 2^2\left[=\frac{2\pi}{3}\text{ or }2.09\right]$	**B1**	SOI AWRT
	[Area of triangle =]$\frac{1}{2}\times 2^2\times\sin\left(\frac{\pi}{3}\right)$ or other valid method $\left[=\sqrt{3}\text{ or }1.73\right]$	**B1**	AWRT Allow use of 60°
	$\left[\text{Area of coin} = 3\text{ segments} + \text{triangle} \Rightarrow\right]$ $3\left(\frac{2\pi}{3}-\sqrt{3}\right)+\sqrt{3}$ [= 2.82]	**M1**	OE Or 3 sectors – 2 triangles $\left(3\times\frac{2\pi}{3}-2\times\sqrt{3}\right)$ or Sector + 2 segments $\left(\frac{2\pi}{3}+2\left(\frac{2\pi}{3}-\sqrt{3}\right)\right)$
	$2\pi-2\sqrt{3}$ or $2(\pi-\sqrt{3})$	**A1**	Must be one of these simplified versions but equivalent decimal answers can score B1B1M1
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{\cos\theta}{\sin\theta}=\frac{2-\sin\theta}{\cos\theta}$ leading to $\cos^2\theta[\sin\theta]=\sin\theta(2-\sin\theta)[\sin\theta]$	***M1**	OE. Forming a correct equation in θ only using the terms of the GP and an attempt to clear fractions.
	$\cos^2\theta+\sin^2\theta=2\sin\theta$ leading to $\sin\theta=\left[\frac{1}{2}\right]$	**DM1**	Correct use of $\cos^2\theta+\sin^2\theta=1$ and attempt to solve for $\sin\theta$.
	$[\theta=]\frac{\pi}{6}$ or 0.524	**A1**	AWRT A0 for $\theta=30°$. Condone inclusion of $\frac{5\pi}{6}$ and/or
		3	

Question	Answer	Marks	Guidance
5(b)	$a = \frac{1}{2} \quad r = \sqrt{3}$	**B1**	OE SOI Trigonometric values need to have been evaluated but allow decimal equivalents (0.5 and 1.73 AWRT)
	$S_{10} = \sin\left(their\frac{\pi}{6}\right)\left(\frac{1-\left(their\sqrt{3}\right)^{10}}{1-\left(their\sqrt{3}\right)}\right)$	**M1**	Use of a correct formula for S_{10}, with *their* value of θ. Their $\sqrt{3}$ needs to come from $\frac{\cos(their\theta)}{\sin(their\theta)}$ or $\frac{2-sin(their\theta)}{cos(their\theta)}$ OE
	$[S_{10} =]\frac{121}{\sqrt{3}-1}$	**A1**	$\frac{-121}{1-\sqrt{3}}$ or 165 AWRT scores B1M1A0**.**
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{d}{dx}\left(x^2-8x+5\right)=0\ \left[2x-8=0\right]$	**M1**	Correct differentiation of x^2 and equating their $\frac{dy}{dx}$ to 0.
	Alternative method 1 for first mark of Question 6(a)		
	$y=\left(x-4\right)^2-11$	**M1**	Attempt to complete the square as far as $y=\left(x-4\right)^2\pm k$.
	Alternative method 2 for first mark of Question 6(a)		
	$x=\frac{-b}{2a}=\frac{\pm 8}{2}$	**M1**	
	$x=4, y=-11$	**A1**	Answers from $x=\frac{8\pm\sqrt{64-20}}{2}$ leading to $x=4\pm\sqrt{11}$ scores M0A0
		2	
6(b)	$x=\left(\textit{their } x \text{ value from } a\right)+4$ [=8]	**B1 FT**	Can be from finding the equation of the transformed curve, differentiating and putting $\frac{dy}{dx}=0$.
	$y=\left\{\left(\textit{their } y \text{ value from } a\right)\times 2\right\}+1$ [– 21]	**B1 FT**	Can be from putting x = 8 in the equation of the transformed curve.
		2	If B0B0 scored, **SC B1** for sight of $\left(4,-22\right)$.

Question	Answer	Marks	Guidance
6(c)	$2\left(x^2-8x+5\right)$ or $2\{\left(x-4\right)^2-11\}$	**B1**	Can be implied if both transformations done together: $2\left(\left(x-4\right)^2-8\left(x-4\right)+5\right)+1$ OE.
	$\left(\left(x-4\right)^2-8\left(x-4\right)+5\right)+1$ or $\left\{\left(x-4-4\right)^2-their11\right\}+1$	**M1**	For the *x* translation, each *x* becomes $\left(x-4\right)$.
		M1	For the y translation of +1.
	$y=2x^2-32x+107$ or $a=2, b=-32, c=107$	**A1**	Evidence to support *their* answer may be in **(b)** but answer must be seen in **(c).**
		4	

Question	Answer	Marks	Guidance
7(a)	$\left(2x-1\right)\left(4x^2+2x-1\right)=8x^3+4x^2-2x-4x^2-2x+1=8x^3-4x+1$	**B1**	AG Six correct terms leading to the correct answer.
		1	

Question	Answer	Marks	Guidance
7(b)	Starting with the LHS $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the numerator and denominator.
	$=\dfrac{1}{1-\cos^2\theta-\cos^2\theta}$ need to see clear evidence of this step	DM1	For use of $\sin^2\theta+\cos^2\theta=1$ twice, in a correct expression, resulting in an expression in $\cos^2\theta$.
	$=\dfrac{1}{1-2\cos^2\theta}$	A1	AG
	Alternative method 1 for Question 7(b)		
	Starting with the RHS $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta-2\cos^2\theta}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\sin^2\theta+\cos^2\theta=1$ twice.
	$=\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ need to see clear evidence of this step	DM1	Dividing throughout by $\cos^2\theta$.
	$=\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$	A1	AG

Question	Answer	Marks	Guidance
7(b)	**Alternative method 2 for Question 7(b)**		
	Starting with the LHS $\dfrac{\sec^2\theta}{\sec^2\theta - 2}$	***M1**	For use of 1+ $\tan^2\theta = \sec^2\theta$ twice.
	Clear statement $\Rightarrow \dfrac{1}{1-2\cos^2\theta}$	**DM1**	AG For multiplying throughout by $\cos^2\theta$ to give the RHS.
		A1	
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{1}{1-2\cos^2\theta}=4\cos\theta$ leading to $1=\ 4\cos\theta\left(1-2\cos^2\theta\right)$ $\left[8\cos^3\theta-4\cos\theta+1=0\right]$	**B1**	Replace LHS with RHS from **(b)** and clear fractions.
	$\left(2\cos\theta-1\right)\left(4\cos^2\theta+2\cos\theta-1\right)[=0]$	***B1**	Use of the expression from **(a)** with $x=\cos\theta$.
	$\left[x \text{ or } \cos\theta=\right]\dfrac{1}{2}\text{and}\dfrac{-2\pm\sqrt{4+16}}{8}$ OR $0.31, -0.81$ AWRT	**DB1**	OE For all three values.
	$\left[\theta=\right]60°, 72°, 144°$	**B2,1,0**	B2 for three correct answers only, B1 for two correct answers and no others (but allow $36°$ instead of $144°$) in the given range or 3 correct answers plus other values in the given range. Ignore answers outside of the given range. Accept AWRT 72.0, 144.0 . **SC B1** for all 3 correct answers in radians and no others: $\dfrac{\pi}{3}, \dfrac{2\pi}{5}$ and $\dfrac{4\pi}{5}$.
		5	

Question	Answer	Marks	Guidance
8(a)	$y=(x+a)^2-a$ leading to $(x+a)^2=y\pm a$	*M1	x and y may be interchanged initially. Allow ± errors for these method marks.
	$x=[\pm]\sqrt{y\pm a}\pm a$	DM1	
	Alternative method for first 2 marks of Question 8(a)		
	$x=(y+a)^2-a$ leading to $y^2+2ay+a^2-a-x[=0]$	*M1	Allow ± errors for this method mark.
	$y=\dfrac{-2a\pm\sqrt{4a^2-4\left(a^2-a-x\right)}}{2}$	DM1	
	[y or $f^{-1}(x)$ =]$-\sqrt{x+a}-a$	A1	OE Must choose negative root.
		3	
8(b)(i)	$x\geqslant -a$	B1	Ignore infinity limit if included.
		1	
8(b)(ii)	y or $f^{-1}[(x)]\leqslant -a$	B1	Ignore negative infinity limit if included.
		1	

Question	Answer	Marks	Guidance
8(c)	$\left[\mathrm{gf}\left(\frac{7}{2}\right)=\right]2\left(\left(x+\frac{7}{2}\right)^2-\frac{7}{2}\right)-1$ or $2x^2+4\left(\frac{7}{2}\right)x+2\left(\frac{7}{2}\right)^2-2\left(\frac{7}{2}\right)-1[=0]$	**B1**	OE Alternatively, $[\mathrm{gf}(x)=0\Rightarrow]\mathrm{f}(x)=\frac{1}{2}$.
	$[x=]-\frac{7}{2}\pm 2$ or $\frac{-14\pm\sqrt{14^2-4\times 2\times\frac{33}{2}}}{4}$ $\left[\frac{-14\pm\sqrt{64}}{4}\right]$ or factorising	**M1**	OE Solving their three term quadratic equation as far as two solutions or correctly selecting the negative root only. Alternatively, $\pm\sqrt{\frac{1}{2}+\frac{7}{2}}-\frac{7}{2}$.
	$[x=]-\frac{11}{2}$	**A1**	If B1M0 scored then award **SCB1** for the correct final answer.
		3	

Question	Answer	Marks	Guidance
9(a)	$2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}} = 3x^{-\frac{1}{2}} + 12$ all $\times x^{\frac{1}{2}}$ $\Rightarrow x - 6x^{\frac{1}{2}} + 5 = 0$	***M1**	OE Equating the two expressions in x and then multiplying each term by $x^{\frac{1}{2}}$ or by their substitution for $x^{\frac{1}{2}}$. Coefficients need to be retained but condone +/– sign errors. Allow $x^{\frac{1}{2}}$ replaced by x.
	$\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} - 5\right)$ [= 0] or [x=] $\dfrac{6 \pm \sqrt{36 - 4\times1\times5}}{2}$	**DM1**	OE Solving their three-term quadratic.
	Alternative method for first 2 marks of Question 9(a)		
	$2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}} = 3x^{-\frac{1}{2}} + 12$ all $\times x^{\frac{1}{2}}$ leading to $2x + 10 = 12x^{\frac{1}{2}}$	***M1**	Equating the two expressions in x and isolating their term in $x^{\frac{1}{2}}$.
	$(2x+10)^2 = 144x$ leading to [4]$\left(x^2 - 26x + 25\right)$[= 0] leading to [4]$(x-25)(x-1)$ [= 0] or [x=] $\dfrac{26 \pm \sqrt{676 - 4\times1\times25}}{2}$	**DM1**	OE Squaring both sides, rearranging and solving a three-term quadratic.
	$x = 1$ and 25 , $y = 15$ and $12\frac{3}{5}$	**A1, A1**	A1 for both x-values and A1 for both y values. If M1DM0 scored then **SCB1B1** is available for final answers.
		4	Answers without working score 0/4

Question	Answer	Marks	Guidance
9(b)	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)-\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[dx]$ $\left[=-2x^{\frac{1}{2}}+12-10x^{-\frac{1}{2}}\right]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{-\dfrac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+12x\left\{-\dfrac{10x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	B1 for either { }. B1 for completely correct integration of their expression following through +/– sign errors from the subtraction.
	$\left(-\frac{4}{3}(\textit{their}\ 25)^{\frac{3}{2}}+12(\textit{their}\ 25)-20(\textit{their}\ 25)^{\frac{1}{2}}\right)-$ $\left(-\frac{4}{3}(\textit{their}\ 1)^{\frac{3}{2}}+12(\textit{their}\ 1)-20(\textit{their}\ 1)^{\frac{1}{2}}\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in *their* integrated expression, defined by at least one correct fractional power, and subtraction.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for first 4 marks of Question 9(b)**		
	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)[\mathrm{dx}]-\int\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[\mathrm{d}x]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}+12x\right\}[-]\left\{\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{13x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	OE One mark for each correct expression.
	$\left(\left(6(\textit{their}\ 25)^{\frac{1}{2}}+12(\textit{their}\ 25)\right)-\left(6(\textit{their}\ 1)^{\frac{1}{2}}+12(\textit{their}\ 1)\right)\right)$ [−] $\left(\left(\frac{4}{3}(\textit{their}\ 25)^{\frac{3}{2}}+26(\textit{their}\ 25)^{\frac{1}{2}}\right)-\left(\frac{4}{3}(\textit{their}\ 1)^{\frac{3}{2}}+26(\textit{their}\ 1)^{\frac{1}{2}}\right)\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in both of their integrated expressions, defined by at least one correct fractional power, and subtraction.
	[Area =] $\frac{128}{3}, 42\frac{2}{3}$, 42.7	**A1**	AWRT If M1B1B1M0 then **SC B1** available for correct final answer. Condone negative answer if corrected.
		5	Condone the presence of π for the first 4 marks but use of $\int \boldsymbol{y}^2$ scores 0/5

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx}=\left\{\frac{5}{3}(4x-3)^{\frac{2}{3}}\right\}\{\times 4\}\left\{-\frac{20}{3}\right\}$	**B2,1,0**	B2 Three correct unsimplified { } and no others. B1 Two correct { }or three correct { } and an additional term e.g. + *c*. B0 More than one error.
	$\left[\frac{20}{3}(4x-3)^{\frac{2}{3}}-\frac{20}{3}=0\right]$ leading to $(4x-3)^2=k,\ k>0$ leading to $4x-3=\pm m$	**M1**	Equating *their* $\frac{dy}{dx}$ to 0 and using a valid method to arrive at 2 answers.
	$[4x-3=\pm 1]\quad [x=]\frac{1}{2},1$	**A1**	
	$\frac{d^2y}{dx^2}=\frac{40}{9}(4x-3)^{-\frac{1}{3}}\times 4$	**B1**	OE
	$\left[x=\frac{1}{2}\right]\ \frac{d^2y}{dx^2}=\left(\frac{160}{9}\right)(4x-3)^{-\frac{1}{3}}<0$ or $-\frac{160}{9}$ or -17.8 so max $[x=1]\ \frac{d^2y}{dx^2}=\left(\frac{160}{9}\right)(4x-3)^{-\frac{1}{3}}>0$ or $\frac{160}{9}$ or 17.8 so min	**B1**	If $\frac{d^2y}{dx^2}$ evaluated the answers for both must be correct OR Clear use of change in sign of $\frac{dy}{dx}$ correctly for both B1's. If B1M1A0B0B0 scored then **SCB1** can be awarded for: $\frac{dy}{dx}=\left\{\frac{5}{3}(4x-3)^{\frac{2}{3}}\right\}-\left\{\frac{20}{3}\right\}$ leading to $(4x-3)^2=64$ leading to $x=-\frac{5}{4},\frac{11}{4}$. $\frac{d^2y}{dx^2}=\frac{10}{9}(4x-3)^{-\frac{1}{3}}$, $x=-\frac{5}{4}$, $\frac{d^2y}{dx^2}<0$ so max, $x=\frac{11}{4}$, $\frac{d^2y}{dx^2}>0$ so min.
		6	

Question	Answer	Marks	Guidance
10(b)	$x < \frac{1}{2},\ x > 1$	B1	Allow $\leqslant$ and/or $\geqslant$. FT only from special case $x < -\frac{5}{4},\ x > \frac{11}{4}$ Condone: $1 < x < \frac{1}{2}$.
		1	

Question	Answer	Marks	Guidance
11(a)	$\left(their\,\frac{7-4}{p-6}\right)\times\left(their\,\frac{18-7}{14-p}\right)=-1$ OR Scalar product leading to $(14-p)(6-p)-33=0$	***M1**	Their gradients must both come from $\frac{\text{Difference in the } y\text{s}}{\text{Difference in the } x\text{s}}$.
	$p^2-20p+84=33$ leading to $p^2-20p+51[=0]$ or $p^2-20p=-51$	**A1**	Clearing of fractions and collecting terms to arrive at the three-term quadratic. Allow integer multiples.
	Alternative method for first 2 marks of Question 11(a)		
	$(p-6)^2+(7-4)^2+(14-p)^2+(18-7)^2=(14-6)^2+(18-4)^2$ OR E.g. $(10-p)^2+4^2=4^2+7^2$	***M1**	For correct use of Pythagoras with *A*,*B* and *C*. OR For correct use of Pythagoras with the centre, B and one of the other two points.
	$2p^2-40p+102[=0]$	**A1**	OE Collecting terms to arrive at the three-term quadratic.
	$[2](p-3)(p-17)$ or $\frac{20\pm\sqrt{20^2-4\times 51}}{2}$	**DM1**	OE Solving their three-term quadratic.
	$p=3$	**A1**	If M1A1DM0 scored then **SC B1** is available for final answer.
		4	

Question	Answer	Marks	Guidance
11(b)	[Midpoint or Centre is] (10, 11)	**B1**	SOI by final answer.
	$\frac{1}{2}\sqrt{(14-6)^2+(18-4)^2}$ or $(18-\textit{their}11)^2+(14-\textit{their}10)^2$ or $(\textit{their}11-4)^2+(\textit{their}10-6)^2$ $\left[r^2=65 \text{ or } r=\sqrt{65}\right]$	**M1**	Finding half of the length of *AC* or using their centre, which cannot be A, B or C, to find r^2 or r. Note: $r=65$ is M0.
	$(x-10)^2+(y-11)^2=65$ or $x^2+y^2-20x-22y+156=0$	**A1**	$(x-6)(x-14)+(y-4)(y-18)=0$ scores 3/3.
		3	
11(c)	$\frac{18-\textit{their}11}{14-\textit{their}10}$ or $\frac{\textit{their}11-4}{\textit{their}10-6}$ or $\frac{18-4}{14-6}$ $\left[=\frac{7}{4}\right]$	***M1**	Gradient of *their* centre, which cannot be A, B or C, from part **(b)**, to A or C or the gradient of AC but working needed if incorrect centre. OR by clearly differentiating and substitution of (14,18).
	$y-18=-\frac{1}{\textit{their}\,\frac{7}{4}}(x-14)$	**DM1**	OE Using (14,18) and $-\frac{1}{\textit{their}\,\frac{7}{4}}$ to form the equation of a straight line.
	$4x+7y-182=0$	**A1**	All terms on one side in any order. Allow multiples of this format by an integer only.
		3	