Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

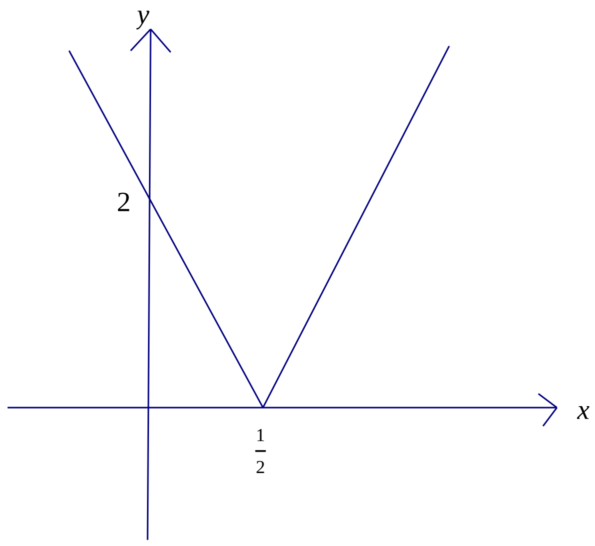
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

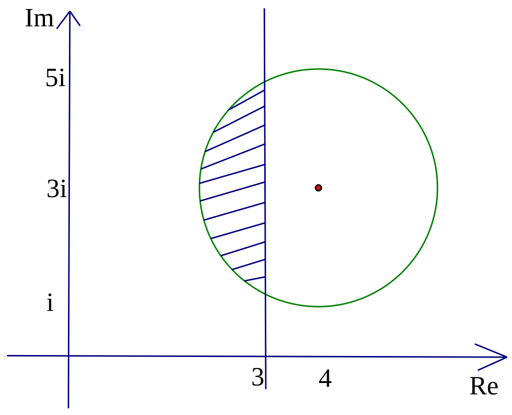
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

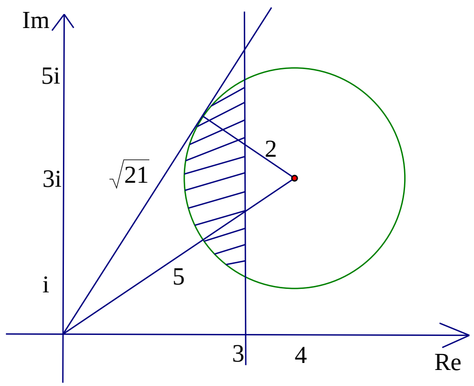
Question	Answer	Marks	Guidance
1(a)		**B1**	Show a recognizable sketch graph of $y = \|4x - 2\|$. Roughly symmetrical. Should extend into the second quadrant. Ignore $y = 4x - 2$ below the axis if intention is clear e.g. dashed or the required lines are clearly bolder. Some indication of scale on **both** axes – accept dashes. Must go beyond (0, 2) and (1, 2). Ignore any attempt to sketch $y = 1 + 3x$.
		1	

Question	Answer	Marks	Guidance
1(b)	Obtain critical value $x=3$	**B1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Solve the linear equation $1+3x=2-4x$	**M1**	Or corresponding linear inequality.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Allow with a comma, or nothing between. Strict inequalities only. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ **and** $x>3$.
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(4x-2)^2>(1+3x)^2$, or corresponding quadratic equation	**M1**	e.g. $7x^2-22x+3=0$. Available if they start with the correct equation / inequality, have a correct method for squaring (i.e. not $(a+b)^2=a^2+b^2$) and a correct method for solving. Need to obtain at least one critical value.
	Obtain critical value $x=3$	**A1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Strict inequalities only. Allow with a comma, or nothing between. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ and $x>3$.
		4	

Question	Answer	Marks	Guidance
2	Obtain $\frac{dx}{dt} = \frac{2}{t}\ln t$	**B1**	Any equivalent form.
	Obtain $\frac{dy}{dt} = -2te^{2-t^2}$	**B1**	Any equivalent form.
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and substitute $t = e$	**M1**	Correct use of chain rule for $\frac{dy}{dx}\left(\frac{-2e^2e^{2-e^2}}{2\ln e}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
	Obtain $-e^{4-e^2}$	**A1**	ISW Accept $-0.0337(405..)$. Accept $-e^4e^{-e^2}$, $\frac{-e^4}{e^{e^2}}$ and $-e^2e^{2-e^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
		4	

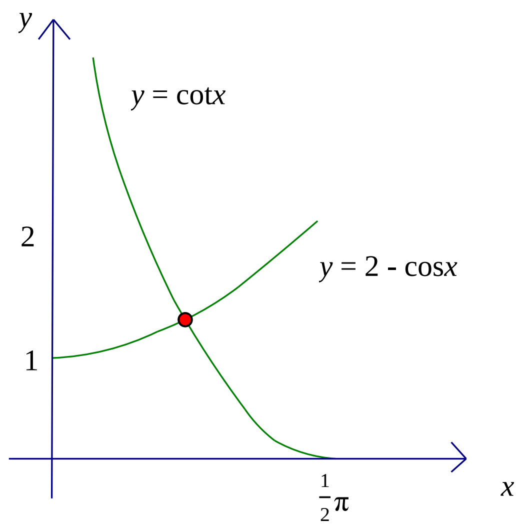
Question	Answer	Marks	Guidance
3	Substitute $x=\frac{1}{2}$ and equate the result to zero	**M1**	Or divide as far as $(2x-1)\left(x^2+px+q\right)$ and equate constant remainder to zero.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $\frac{1}{4}+\frac{a}{4}-\frac{11}{2}+b=0$ or $a+4b=21$.
	Substitute $x=-1$ and equate result to 12	**M1**	Or divide as far as $(x+1)\left(2x^2+rx+s\right)$ and equate constant remainder to 12.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $-2+a+11+b=12$ or $a+b=3$.
	Obtain $a=-3,\quad b=6$	**A1**	
		5	
	$\begin{array}{c\|llll} & & x^2 & \left(\frac{a+1}{2}\right)x & \frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline 2x-1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & -x^2 & & \\ \hline & & (a+1)x^2 & -11x & \\ & & (a+1)x^2 & -\left(\frac{a+1}{2}\right)x & \\ \hline & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & b \\ & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & -\frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline & & & & b-\frac{21}{4}+\frac{a}{4} \\ \hline \end{array}$		$\begin{array}{c\|llll} & & 2x^2 & (a-2)x & (-9-a) \\ \hline x+1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & 2x^2 & & \\ \hline & & (a-2)x^2 & -11x & \\ & & (a-2)x^2 & (a-2)x & \\ \hline & & & (-9-a)x & b \\ & & & (-9-a)x & (-9-a) \\ \hline & & & & b+9+a \\ \hline \end{array}$

Question	Answer	Marks	Guidance
4(a)	Show a circle with centre $4+3\mathrm{i}$. Accept a curved shape with correct point roughly in the middle.	**B1**	
	Show a circle with radius 2 and centre not at the origin. The shape should be consistent with their scales	**B1**	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	**B1**	
	Shade the correct region on a correct diagram Any other shading must be accompanied by words to explain which region is required	**B1**	Need some indication of scale e.g. label the centre, mark key points on the axes or dashes on the axes. Condone dotted lines in place of solid lines Condone correct shaded shape but not an entire circle
		4	

Question	Answer	Marks	Guidance
4(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\tan^{-1}\frac{3}{4}+\sin^{-1}\frac{2}{5}$ $\quad(0.6435+0.4115)$	**M1**	
	Obtain answer 1.06 (accept 1.055 or 1.056) radians or $60.45°$ (accept $60.4°$ or $60.5°$)	**A1**	
	Alternative method for question 4(b)		
	Tangent to circle passing through origin has equation $y=mx$. The equation $(x-4)^2+(y-3)^2=4$ will have one root. Hence $(1+m^2)x^2-(8+6m)x+21=0$, discriminant = 0 = $48m^2-96m+20$ and $m=\dfrac{6\pm\sqrt{21}}{6}$ with the larger value needed to give greatest arg(*z*). Required angle is $\tan^{-1}m$.	**M1**	Complete method for finding the greatest value of arg(*z*).
	Obtain answer 1.06 radians or $60.45°$	**A1**	Accept 1.055 or 1.056 radians. Accept $60.4°$ or $60.5°$.
		2	

Question	Answer	Marks	Guidance
5	Split fraction to obtain $1+\dfrac{x-4}{x^2+4}$	**B1**	
	Attempt integration and obtain $p\ln\left(x^2+4\right)$ or $q\tan^{-1}\left(\dfrac{x}{2}\right)$ from correct working	**M1**	Allow for $p\ln\left(x^2+4\right)$ from $\int\dfrac{x}{x^2+4}\mathrm{d}x$ but only if a correct method for splitting has been used.
	Obtain $\frac{1}{2}\ln\left(x^2+4\right)$	**A1 FT**	Follow through is on their coefficients in the partial fraction. Allow from $\dfrac{x^2}{x^2+4}+\dfrac{x}{x^2+4}$ even if the split of the fraction is not complete. If $1-\dfrac{4}{x^2+4}+\dfrac{x}{x^2+4}$ later seen or implied, award the B1. Only available from a correct split, not from an approach using parts that is incomplete.
	Obtain $-2\tan^{-1}\left(\dfrac{x}{2}\right)$	**A1 FT**	Only available from a correct split, not from an approach using parts that is incomplete.
	Correct use of correct limits 0 and 6 in an expression involving $p\ln\left(x^2+4\right)$, $q\tan^{-1}\left(\dfrac{x}{2}\right)$ and no incorrect terms.	**M1**	p and q should be constants. The x term is not required at this stage.
	Obtain $6+\frac{1}{2}\ln 10-2\tan^{-1}3$	**A1**	ISW Or three term equivalent. (Must combine the ln terms.) Accept with $\frac{1}{2}\ln\lvert 10\rvert$.

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Use the substitution $x = 2\tan\theta$ to obtain $\int 2\tan^2\theta + \tan\theta \, d\theta$	**B1**	
	Attempt integration and obtain $p\tan\theta$ or $r\ln(\sec\theta)$ from correct working	**M1**	
	Obtain $2\tan\theta\,(-2\theta)$ and	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Obtain $\ln\sec\theta$	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Use correct limits 0 and $\tan^{-1}3$ in an expression involving $u\tan\theta$, $v\ln\sec\theta$ and no incorrect terms	**M1**	u and v should be constants. The θ term is not required at this stage.
	Obtain $6 + \ln\left\|\sec\left(\tan^{-1}3\right)\right\| - 2\tan^{-1}3$	**A1**	ISW Or three term equivalent. Not required to simplify $\ln\left\|\sec\left(\tan^{-1}3\right)\right\|$.
		6	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph. e.g. $y = \cot x$: x intercept should be correct. Not touching the y-axis. No incorrect curvature. Ignore anything outside $0 < x \leqslant \frac{1}{2}\pi$.	**B1**	
	Sketch a second relevant graph **and justify the given statement** e.g. $y = 2 - \cos x$: Condone if looks almost straight, but not if drawn with a ruler and not incorrect curvature. Correct y intercept. Needs to be drawn for $0 < x \leqslant \frac{1}{2}\pi$. Ignore outside this.	**B1**	
		2	2nd B1 requires a mark at the point of intersection or a suitable comment for the justification.
6(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.6$ and $x = 0.8$ Must be working in radians. Values correct to at least 2 significant figures. Need all relevant values but only one (pair) needs to be correct to award M1. Complete set of values for their expression. If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	**M1**	e.g. $1.17 < 1.46$, $1.30 > 0.971$, $-0.29 < 0$, $0.33 > 0$. $-0.20 < 0$, $0.342 > 0$ from $\tan x(2 - \cos x) - 1 = 0$. $0.80 < 1$, $1.34 > 1$ from $\tan x(2 - \cos x) = 1$. $0.146 > 0, -0.105 < 0$ from $x - \tan^{-1}\left(\frac{1}{2 - \cos x}\right)$.
	Complete the argument correctly with correct calculated values (awrt 2 s.f.). Clear comparison for their expression. Allow work on a smaller interval.	**A1**	Accept truncated values. If comparing with 0 can either indicate different signs or a negative product.
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once. Must be working in radians	**M1**	
	Obtain final answer 0.68	**A1**	Must be a clear conclusion.
	Show sufficient iterations to at least 4 d.p. to justify 0.68 to 2 d.p. or show there is a sign change in the interval (0.675, 0.685). Allow recovery. Allow truncation. Allow small differences in the 4th s.f.	**A1**	e.g. 0.7, 0.6806, 0.6855, 0.6843, 0.6846 0.6, 0.7053, 0.6792, 0.6858, 0.6842, 0.6846 0.8, 0.6545, 0.6920, 0.6826, 0.6850, 0.6844, 0.6845 .
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct expansion for $\cos(2\theta+\theta)$	***M1**	
	Use correct double angle formulae to express $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$	**DM1**	
	Show sufficient working to confirm $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
7(b)	Use the identity and correct double angle formula to obtain an equation in $\cos\theta$ only. Must come from using all three terms in the given equation.	***M1**	e.g. $4\cos^3\theta - 3\cos\theta + \cos\theta(2\cos^2\theta - 1) = \cos^2\theta$ $6\cos^3\theta - \cos^2\theta - 4\cos\theta = 0$ or $6\cos^2\theta - \cos\theta - 4 = 0$.
	Obtain $\theta = 90°$	**B1**	Allow if $\cos\theta$ obtained correctly as a factor of *their* expression (even if there is an error in the quadratic factor). Can follow M0.
	Solve a 3-term quadratic in $\cos\theta$ to obtain a value of θ	**DM1**	
	Obtain one value e.g. $25.3°$	**A1**	Accept awrt $25.3°$.
	Obtain a second value e.g. $137.5°$ and no extras in range	**A1**	Accept awrt $137.5°$. Ignore values outside the range. Mark solutions in radians as a misread $(0.442, 1.57, 2.40)$.
		5	

Question	Answer	Marks	Guidance
8(a)	Multiply both sides by $a+2\mathrm{i}$ and attempt expansion of right-hand side	***M1**	
	Use of $\mathrm{i}^2=-1$ seen at least once (or implied)	**DM1**	e.g. $2+3a\mathrm{i}=\lambda(2a+2)+\lambda\mathrm{i}(-a+4)$
	Compare real and imaginary parts to obtain an equation in *a* only $[2=\lambda(2a+2),\quad 3a=\lambda(-a+4)]$	**M1**	e.g. $\frac{3a}{2}=\frac{-a+4}{2a+2}$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	**A1**	AG
	Alternative method for question 8(a)		
	Multiply top and bottom of the left-hand side by $a-2\mathrm{i}$ and attempt both expansions	***M1**	Do not need the right-hand side at this stage.
	Use of $\mathrm{i}^2=-1$ seen at least once or implied	**DM1**	e.g. $[\lambda(2-\mathrm{i})=]\frac{8a+\mathrm{i}(3a^2-4)}{a^2+4}$.
	Compare real and imaginary parts to obtain an equation in *a* only	**M1**	e.g. $8a=-2(3a^2-4)$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	**A1**	AG
		4	
8(b)	Solve given quadratic to obtain a value of *a* and use this to form an equation in λ only (based on an equation seen in *their* working in **(a)** or **(b)**)	**M1**	Can be implied by relevant working seen or a correct value for λ seen.
	Obtain $a=-2,\ \lambda=-1$ or $a=\frac{2}{3},\ \lambda=\frac{3}{5}$	**A1**	Allow $\frac{6}{10}$ and 0.6.
	Obtain second correct pair of values	**A1**	
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule	***M1**	As far as $p\cos x\cos 2x + q\sin x\sin 2x$ or full working (u, v, du/dx, dv/dx) shown.
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos x\cos 2x - 2\sin x\sin 2x$	**A1**	OE
	Equate derivative to zero and use correct double angle formulae	**DM1**	Allow if only have one double angle in their derivative.
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$ or equivalent	**A1**	e.g. $\cos x\left(6\cos^2 x-5\right)=0$, $5\tan^2 x=1$. Simplified but not necessarily factorised - like terms must be collected.
	Obtain $a=0.42$	**A1**	Only. Accept $x=0.42$.
	Alternative method for question 9(a)		
	Use correct double angle formula	***M1**	
	Obtain $\sin x - 2\sin^3 x$ or equivalent	**A1**	
	Use correct chain rule or product rule to differentiate and equate the derivative to zero	**DM1**	
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$	**A1**	OE
	Obtain $a=0.42$	**A1**	Only. Accept $x=0.42$.
		5	

Question	Answer	Marks	Guidance
9(b)	Use double angle formula and obtain $p\cos^3 x + q\cos x$ correctly	*M1	e.g. from $\int 2\cos^2 x\sin x - \sin x\,dx$.
	Obtain $\pm\left(-\frac{2}{3}\cos^3 x + \cos x\right)$	A1	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\left(-\frac{2}{3}\left[\left(\frac{-1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3\right] - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.
	Alternative method 1 for question 9(b)		
	Use integration by parts **twice** and obtain $r\cos x\cos 2x + s\sin x\sin 2x$	*M1	Seen, not just implied.
	Obtain $\frac{1}{3}\cos x\cos 2x + \frac{2}{3}\sin x\sin 2x$	A1	Accept $\pm$ (correct for *their* integral).
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\frac{1}{3}\left(0 + 2\times\frac{1}{\sqrt{2}}\times -1 - 0 - 2\times\frac{1}{\sqrt{2}}\times 1\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.

Question	Answer	Marks	Guidance
	Alternative method 2 for question 9(b)		
	Use factor formula and integrate to obtain $g\cos 3x + h\cos x$	***M1**	$\int \frac{1}{2}(\sin 3x - \sin x)\mathrm{d}x$.
	Obtain $\pm\left(-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x\right)$	**A1**	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	**DM1**	OE $\mp\frac{1}{\sqrt{2}}\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{6}+\frac{1}{2}\right)$.
	Obtain $\dfrac{2\sqrt{2}}{3}$	**A1**	Or exact equivalent. Final answer must be positive.
		4	

Question	Answer	Marks	Guidance
10(a)	Use the correct process to calculate the scalar product of the direction vectors	**M1**	$(-2+4+2c)$.
	Divide the scalar product by the product of the moduli and equate the result to $\cos 60°$	**M1**	Or equivalent e.g. $2+2c=\sqrt{6}\sqrt{20+c^2}\cos 60°$. Allow for the correct process using $60°$ but the wrong vectors.
	Obtain correct equation in c	**A1**	e.g. $\dfrac{2+2c}{\sqrt{6}\sqrt{20+c^2}}=\dfrac{1}{2}$ or $10c^2+32c-104=0$.
	Obtain $c=2$	**A1**	Only.
		4	

Question	Answer	Marks	Guidance
10(b)	Calling $(6,-3,6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix}-3+\lambda\\1+\lambda\\-5+2\lambda\end{pmatrix}$.
	Equate the scalar product of *their* $\overrightarrow{AP}$ and a direction vector for *l* to zero and obtain an equation in λ	***M1**	e.g. $(-3+\lambda)+(1+\lambda)+(-10+4\lambda)=0$.
	Solve and obtain $\lambda=2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 1 for question 10(b)		
	Calling $(6,-3,6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix}-3+\lambda\\1+\lambda\\-5+2\lambda\end{pmatrix}$
	Differentiate the modulus of $\overrightarrow{AP}$ or the square of the modulus and equate the derivative to zero	***M1**	e.g. $2(-3+\lambda)+2(1+\lambda)+4(-5+2\lambda)=0$
	Solve and obtain $\lambda=2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$
	Obtain $\sqrt{11}$ from correct working	**A1**	AG

Question	Answer	Marks	Guidance
	Alternative method 2 for question 10(b)		
	Vector from $(6, -3, 6)$ to $(3, -2, 1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	The method works for vector from $(6, -3, 6)$ to any point on *l*.
	Use scalar product to find the angle between *their* vector and the direction of *l*	**M1**	
	Obtain $\cos\theta = \frac{3-1+10}{\sqrt{35}\sqrt{6}}\left(=\sqrt{\frac{24}{35}}\right)$ or $\sin\theta = \sqrt{\frac{11}{35}}$	**A1**	
	Correct use of trig to find the projection of their vector on the normal to *l*	**M1**	$\sqrt{35}\sin\theta = \sqrt{35}\times\sqrt{\frac{11}{35}}$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 3 for question 10(c)		
	Vector from $(6, -3, 6)$ to $(3, -2, 1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	
	Find the vector product of *their* vector and the direction of *l*	**M1**	
	Obtain $\mathbf{i}(2+5)-\mathbf{j}(-6+5)+\mathbf{k}(-3-1)\,(=7\mathbf{i}+\mathbf{j}-4\mathbf{k})$	**A1**	
	Correct use of trig to find the perpendicular distance	**M1**	$\frac{\lvert \textit{their}\text{ vector product}\rvert}{\lvert\text{direction vector}\rvert}$.
	Distance $=\frac{\sqrt{66}}{\sqrt{6}}=\sqrt{11}$	**A1**	
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Question	Answer	Marks	Guidance
11(a)	Correct separation of variables.	**B1**	$\int \frac{1}{y^2+y}\,dy = \int -\frac{1}{x^2}\,dx$. Condone missing integral signs or missing dx, dy, but not both.
	Obtain $\frac{1}{x}$	**B1**	
	Express $\frac{1}{y^2+y}$ in partial fractions or express the denominator of the fraction as a difference of two squares	***M1**	Allow for the correct split of $\frac{\pm 1}{\left(y^2 \pm y\right)}$.
	Obtain $\frac{1}{y}-\frac{1}{y+1}$ or $\frac{1}{\left(y+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$	**A1**	Allow if coefficients for the partial fractions are correct but followed by an error.
	Obtain $\ln y-\ln(y+1)$	**A1**	Or equivalent, dependent on where they left the minus sign.
	Use $x=1, y=1$ to find constant of integration or as limits in a definite integral in an expression containing terms of the form $\frac{p}{x}, q\ln y$ and $r\ln(1+y)$	**DM1**	$\ln\frac{1}{2}=1+C$ If they rearrange the equation before finding the constant of integration then the constant must be of the correct form.
	Correct equation in x and y	**A1**	$\ln\frac{y}{1+y}=\frac{1}{x}-1+\ln\frac{1}{2}$.
	Obtain $y=\frac{e^{\frac{1}{x}-1}}{2-e^{\frac{1}{x}-1}}$	**A1**	Or equivalent e.g. $y=\frac{1}{2e^{1-\frac{1}{x}}-1}$, $y=\frac{1}{e^{1-\frac{1}{x}+\ln 2}-1}$. Accept with decimal value for e^{-1}.
		8	

Question	Answer	Marks	Guidance
11(b)	State that y approaches $\dfrac{1}{2\mathrm{e}-1}$	**B1 FT**	Or exact equivalent. Condone $y=\dfrac{1}{2\mathrm{e}-1}$. FT on an expression in $\mathrm{e}^{\frac{1}{x}}$.
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