

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

MATHEMATICS 9709/31

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

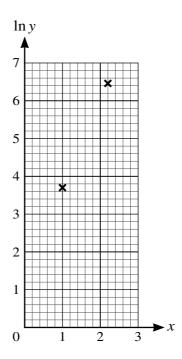
- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

is equal to	<i>3</i> 0.							
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On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z-2i| \le |z+2-i|$ and $0 \le \arg(z+1) \le \frac{1}{4}\pi$. [4]



The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b .	[4]

(a)	Express u in the Cartesian form $x + iy$, where x and y are in terms of a .	[3]
		•••••
(b)	Given that $\arg u = \frac{1}{4}\pi$, find the value of a.	[2]
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5	(a)	Given	that
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$\sin(x+\frac{1}{6}\pi)$	$-\sin(x-$	$\frac{1}{6}\pi\big) = \cos\big($	$(x+\frac{1}{3}\pi)-c\alpha$	$\cos\left(x-\frac{1}{3}\pi\right),$		
find the exact value of $\tan x$.						[4]
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	7
(b)	Hence find the exact roots of the equation
	$\sin(x + \frac{1}{6}\pi) - \sin(x - \frac{1}{6}\pi) = \cos(x + \frac{1}{3}\pi) - \cos(x - \frac{1}{3}\pi)$
	for $0 \le x \le 2\pi$.

	6	The	parametric	equations	of a	curve	are
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$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for t > 0.

(a)	Obtain a simplified expression for $\frac{dy}{dx}$	$\frac{y}{x}$ in terms of t .	[3]
(b)	Hence find the exact coordinates of is -2.	the point on the curve at which the gradi	ent of the normal [3]
			•••••

7 The variables x and θ satisfy the differential equation
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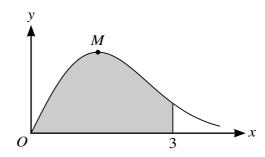
$$\frac{x}{\tan\theta} \frac{\mathrm{d}x}{\mathrm{d}\theta} = x^2 + 3.$$

It is given that x = 1 when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ .	[7]
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	$\sqrt{x} = e^x - 3$	
	has only one root.	[2]
<i>a</i> .		
(b)	Show by calculation that this root lies between 1 and 2.	[2]
(b)	Show by calculation that this root lies between 1 and 2.	
(b)		
(b)		[2]
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(b)		

(c)	Show that, if a sequence of values given by the iterative formula	
	$x_{n+1} = \ln(3 + \sqrt{x_n})$	
	converges, then it converges to the root of the equation in (a).	[1]
(d)	Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of eiteration to 4 decimal places.	each [3]
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The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \ge 0$, and its maximum point M.

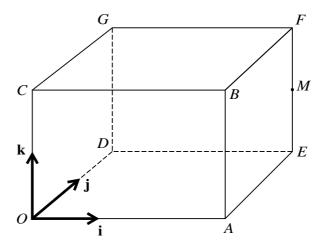
(a)	Find the exact coordinates of M .	[4]
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bounded by the curve, the <i>x</i> -axis and the line $x = 3$.	
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10	Let $f(x) =$	24x + 13		
10	Let $\Gamma(X) =$	$\frac{(1-2x)(2+x)^2}{(1-2x)(2+x)^2}$		

(a)	Express $f(x)$ in partial fractions.	[5]

Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and in	[5]
State the set of values of x for which the expansion in (b) is valid.	[1]



In the diagram, OABCDEFG is a cuboid in which OA = 3 units, OC = 2 units and OD = 2 units. Unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are parallel to OA, OD and OC respectively. M is the midpoint of EF.

(a)	Find the position vector of M .	[1]
The	position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.	
(b)	Calculate angle <i>PAM</i> .	[4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.			
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