Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7079584696

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 **(a)** A random variable X has the distribution Po(25).

Use the normal approximation to the Poisson distribution to find $P(X > 30)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) A random variable Y has the distribution $B(100, p)$ where $p < 0.05$.

Use the Poisson approximation to the binomial distribution to write down an expression, in terms of p, for $P(Y < 3)$. [2]

.....

.....

.....

.....

.....

.....

.....

2 The length, in minutes, of mathematics lectures at a certain college has mean μ and standard deviation 8.3.

(a) The total length of a random sample of 85 lectures was 4590 minutes.

Calculate a 95% confidence interval for μ. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The length, in minutes, of history lectures at the college has mean m and standard deviation s.

(b) Using a random sample of 100 history lectures, a 95% confidence interval for m was found to have width 2.8 minutes.

Find the value of s. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 A researcher read a magazine article which stated that boys aged 1 to 3 prefer green to orange. It claimed that, when offered a green cube and an orange cube to play with, a boy is more likely to choose the green one.

The researcher disagrees with this claim. She believes that boys of this age are equally likely to choose either colour. In order to test her belief, the researcher carried out a hypothesis test at the 5% significance level. She offered a green cube and an orange cube to each of 10 randomly chosen boys aged 1 to 3, and recorded the number, X, of boys who chose the green cube.

Out of the 10 boys, 8 boys chose the green cube.

(a) **(i)** Assuming that the researcher's belief that either colour cube is equally likely to be chosen is valid, a student correctly calculates that $P(X = 8) = 0.0439$, correct to 3 significant figures. He says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

.....

.....

.....

.....

(ii) Carry out the test on the researcher's claim that either colour cube is equally likely to be chosen. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Another researcher claims that a Type I error was made in carrying out the test.

Explain why this cannot be true. [1]

.....

.....

.....

.....

A similar test, at the 5% significance level, was carried out later using 10 other randomly chosen boys aged 1 to 3.

(c) Find the probability of a Type I error. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

4 The height H, in metres, of mature trees of a certain variety is normally distributed with standard deviation 0.67. In order to test whether the population mean of H is greater than 4.23, the heights of a random sample of 200 trees are measured.

(a) Write down suitable null and alternative hypotheses for the test. [1]

.....

.....

.....

The sample mean height, $\bar{h}$ metres, of the 200 trees is found and the test is carried out. The result of the test is to reject the null hypothesis at the 5% significance level.

(b) Find the set of possible values of $\bar{h}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Ajit said, 'In **(b)** we had to assume that $\overline{H}$ is normally distributed, so it was necessary to use the Central Limit Theorem.'

Explain whether you agree with Ajit. [1]

.....

.....

.....

.....

5 The random variable X has probability density function, f, given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a < x < b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(a) It is given that $E(X) = \ln 2$.

Show that $b = 2a$. [3]

(b) Show that $a = \frac{1}{2}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the median of X. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 A factory makes loaves of bread in batches. One batch of loaves contains X kilograms of dried yeast and Y kilograms of flour, where X and Y have the independent distributions $N(0.7, 0.02^2)$ and $N(100.0, 3.0^2)$ respectively.

Dried yeast costs \$13.50 per kilogram and flour costs \$0.90 per kilogram. For making one batch of bread the total of all other costs is \$55. The factory sells each batch of bread for \$200.

Find the probability that the profit made on one randomly chosen batch of bread is greater than \$40. [7]

7 A random variable X has the distribution Po(2.4).

(a) Find $P(2 \leqslant X < 4)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Two independent values of X are chosen.

Find the probability that both of these values are greater than 1. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) It is given that $P(X = r) < P(X = r + 1)$.

(i) Find the set of possible values of r. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Hence find the value of r for which $P(X = r)$ is greatest. [1]

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE