Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

5552004552

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the quotient and remainder when $x^4 - 3x^3 + 9x^2 - 12x + 27$ is divided by $x^2 + 5$. [3]

2 **(a)** Find the coefficient of x^2 in the expansion of $(2x-5)\sqrt{4-x}$. [4]

(b) State the set of values of x for which the expansion in part **(a)** is valid. [1]

3 It is given that $z = -\sqrt{3} + \mathrm{i}$.

(a) Express z^2 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

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(b) The complex number ω is such that $z^2\omega$ is real and $\left|\frac{z^2}{\omega}\right| = 12$.

Find the two possible values of ω, giving your answers in the form $R\mathrm{e}^{\mathrm{i}\alpha}$, where $R > 0$ and $-\pi < \alpha \leqslant \pi$. [3]

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4 The positive numbers p and q are such that

$$\ln\left(\frac{p}{q}\right) = a \quad \text{and} \quad \ln\left(q^2 p\right) = b.$$

Express $\ln\left(p^7 q\right)$ in terms of a and b. [4]

5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z-4-2\mathrm{i}| \leqslant 3$ and $|z| \geqslant |10-z|$. [4]

(b) Find the greatest value of $\arg z$ for points in this region. [2]

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6 The equation of a curve is $2y^2 + 3xy + x = x^2$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2x - 3y - 1}{4y + 3x}$. [4]

(b) Hence show that the curve does **not** have a tangent that is parallel to the x-axis. [3]

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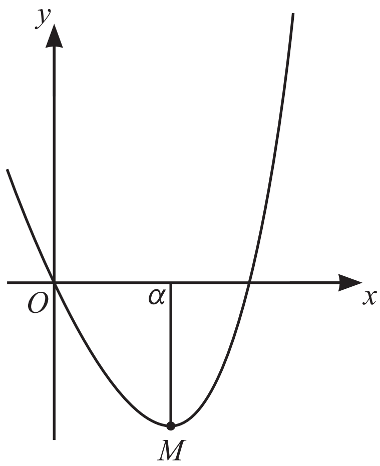

The diagram shows the curve $y = xe^{2x} - 5x$ and its minimum point M, where $x = \alpha$.

(a) Show that α satisfies the equation $\alpha = \frac{1}{2}\ln\left(\frac{5}{1+2\alpha}\right)$. [3]

(b) Verify by calculation that α lies between 0.4 and 0.5 . [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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8 **(a)** Express $3\sin x + 2\sqrt{2}\cos\left(x + \frac{1}{4}\pi\right)$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. State the exact value of R and give α correct to 3 decimal places. [4]

(b) Hence solve the equation

$$6\sin\tfrac{1}{2}\theta + 4\sqrt{2}\cos\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\pi\right) = 3$$

for $-4\pi < \theta < 4\pi$. [5]

9 Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 8\mathbf{i} + 2\mathbf{j} - 6\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(a) Show that $OABC$ is a rectangle. [4]

(b) Use a scalar product to find the acute angle between the diagonals of *OABC*. [4]

10 Let $f(x) = \dfrac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_{-a}^{a} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $p\ln q + r\ln s$ where p and r are integers and q and s are prime numbers. [5]

11 The variables y and θ satisfy the differential equation

$$(1+y)(1+\cos 2\theta)\frac{dy}{d\theta} = e^{3y}.$$

It is given that $y = 0$ when $\theta = \frac{1}{4}\pi$.

Solve the differential equation and find the exact value of $\tan\theta$ when $y = 1$. [9]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.