

Cambridge International AS & A Level

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289478904

MATHEMATICS 9709/12

Paper 1 Pure Mathematics 1

May/June 2024

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

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| Find the value of the positive constant <i>a</i> . | | | | |
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| | vacuation $\frac{7 \tan \theta}{\cos \theta} + 12 = 0$ can be expressed as $12 \sin^2 \theta - 7 \sin \theta - 12 = 0.$ | ro- |
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| | $12\sin \theta - /\sin \theta - 12 = 0.$ | [3] |
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| Hence solve the e | equation $\frac{7 \tan \theta}{\cos \theta} + 12 = 0$ for $0^{\circ} \le \theta \le 360^{\circ}$. | [3] |
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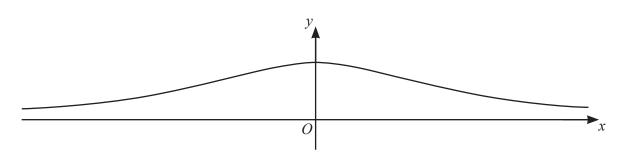
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| 4 | I ne | function | 1 f 1S | defined | as follows |

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

| (a) | Find an expression for $f^{-1}(x)$. | [1] |
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The diagram shows the graph of y = g(x) where $g(x) = \frac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

| b) | State the range of g and explain whether g^{-1} exists. | [2 |
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The function h is defined by $h(x) = \frac{1}{x^2 + 2}$ for $x \ge 0$.

| integ | gers. | on $hf(x) = f$ | (16). Giv | c your an | swer in ur | c form a r | byc, wher | c u, v and |
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5

The first and second terms of an arithmetic progression are $\tan \theta$ and $\sin \theta$ respectively, where

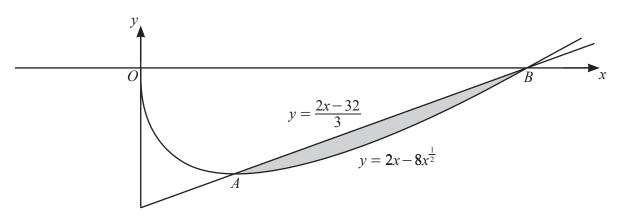
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The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0<\theta<\frac{1}{2}\pi$.

| (b) (i) | Find the sum to infinity of the progression in terms of θ . | [2] |
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| (ii) | Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression | . Give your answer |
| | correct to 3 significant figures. | [3] |
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| F1 | and the coordinates of A and B . | |
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(b)



The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB. It is given that the equation of AB is $y = \frac{2x - 32}{3}$.

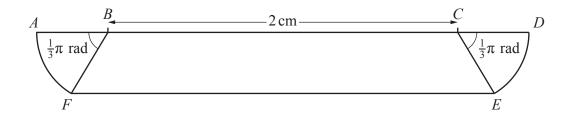
| Find the area of the shaded region between the curve and the line. | [5] |
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The equation of a circle is $(x-6)^2 + (y+a)^2 = 18$. The line with equation y = 2a - x is a tangent to the

| Find the two possible values of the constant <i>a</i> . |
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| for the greater value of a , find the equation of the diameter which is pangent. | [3] |
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The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius r cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that r = 0.4 cm.

| (i) | Show that the length $EF = 2.4 \mathrm{cm}$. | [2] |
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| (ii) | Find the area of the plate. Give your answer correct to 3 significant figures. | [4] |
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| T4 | is since instead that the manimentary of the plate is Com- | |
| 11 | is given instead that the perimeter of the plate is 6 cm. | |
| E: | 1.1. 1 C. C' | |
| F11 | nd the value of r . Give your answer correct to 3 significant figures. | [|
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| Determine the set of values of x for which $f(x)$ is | is decreasing. | |
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| 10 | The equation of a | curve is $y = ($ | $(5-2x)^{\frac{1}{2}}+5$ | for $x < \frac{3}{2}$. |

| | 5 units per second. |
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| I | Find the rate at which the x-coordinate of point P is increasing when $y = 32$. |
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Additional page

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