Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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7805754427

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A bus station has exactly four entrances. In the morning the numbers of passengers arriving at these entrances during a 10-second period have the independent distributions Po(0.4), Po(0.1), Po(0.2) and Po(0.5).

Find the probability that the total number of passengers arriving at the four entrances to the bus station during a randomly chosen 1-minute period in the morning is more than 3. [3]

2 The random variable X has the distribution $\mathrm{N}(31.2, 10.4^2)$. Two independent random values of X, denoted by X_1 and X_2, are chosen.

Find $\mathrm{P}(X_1 > 3X_2)$. [5]

3 The time taken in minutes for a certain daily train journey has a normal distribution with standard deviation 5.8 . For a random sample of 20 days the journey times were noted and the mean journey time was found to be 81.5 minutes.

(a) Calculate a 98% confidence interval for the population mean journey time. [3]

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A student was asked for the meaning of this confidence interval. The student replied as follows.

'The times for 98% of these journeys are likely to be within the confidence interval.'

(b) Explain briefly whether this statement is true or not. [1]

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Two independent 98% confidence intervals are found.

(c) Given that at least one of these intervals contains the population mean, find the probability that both intervals contain the population mean. [2]

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4 **(a)** A random sample of 8 boxes of cereal from a certain supplier was taken. Each box was weighed and the masses in grams were as follows.

261 249 259 252 255 256 258 254

Find unbiased estimates of the population mean and variance. [3]

(b) The supplier claims that the mean mass of boxes of cereal is 253 g. A quality control officer suspects that the mean mass is actually more than 253 g. In order to test this claim, he weighs a random sample of 100 boxes of cereal and finds that the total mass is 25 360 g.

(i) Given that the population standard deviation of the masses is 3.5 g, test at the 5% significance level whether the population mean mass is more than 253 g. [5]

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An employee says, 'This test is invalid because it uses the normal distribution, but we do not know whether the masses of the boxes are normally distributed.'

(ii) Explain briefly whether this statement is true or not. [1]

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5 Sales of cell phones at a certain shop occur singly, randomly and independently.

(a) State one further condition that must be satisfied for the number of sales in a certain time period to be well modelled by a Poisson distribution. [1]

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The average number of sales per hour is 1.2 .

Assume now that a Poisson distribution is a suitable model.

(b) Find the probability that the number of sales during a randomly chosen 12-hour period will be more than 12 and less than 16. [3]

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(c) Use a suitable approximating distribution to find the probability that the number of sales during a randomly chosen 1-month period (140 hours) will be less than 150. [4]

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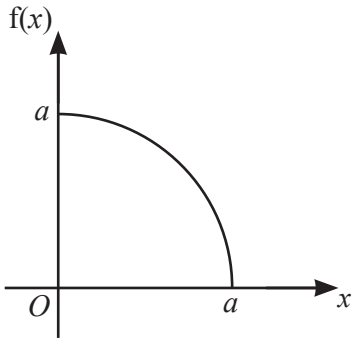

The diagram shows the graph of the probability density function, f , of a random variable X . The graph is a quarter circle entirely in the first quadrant with centre $(0, 0)$ and radius a, where a is a positive constant. Elsewhere $\mathrm{f}(x) = 0$.

(a) Show that $a = \dfrac{2}{\sqrt{\pi}}$. [2]

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(b) Show that $\mathrm{f}(x) = \sqrt{\dfrac{4}{\pi} - x^2}$. [2]

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(c) Show that $\mathrm{E}(X) = \dfrac{8}{3\sqrt{\pi^3}}$. [4]

7 Every July, as part of a research project, Rita collects data about sightings of a particular kind of bird. Each day in July she notes whether she sees this kind of bird or not, and she records the number X of days on which she sees it. She models the distribution of X by $\mathrm{B}(31, p)$, where p is the probability of seeing this kind of bird on a randomly chosen day in July.

Data from previous years suggests that $p = 0.3$, but in 2022 Rita suspected that the value of p had been reduced. She decided to carry out a hypothesis test.

In July 2022, she saw this kind of bird on 4 days.

(a) Use the binomial distribution to test at the 5% significance level whether Rita's suspicion is justified. [5]

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In July 2023, she noted the value of X and carried out another test at the 5% significance level using the same hypotheses.

(b) Calculate the probability of a Type I error. [2]

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Rita models the number of sightings, Y, per year of a different, very rare, kind of bird by the distribution B(365, 0.01).

(c) (i) Use a suitable approximating distribution to find $P(Y = 4)$. [3]

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(ii) Justify your approximating distribution in this context. [1]

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Additional page

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