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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 0 7 5 9 5 7 4 0 8 2 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$. When $p(x)$ is divided by $(x - 4)$ the remainder is equal to 3 times the remainder when $p(x)$ is divided by $(x - 2)$.

Find the values of a and b. [5]

2 Find the exact value of $\int_1^3 x^2 \ln 3x \, dx$. Give your answer in the form $a \ln b + c$, where a and c are rational and b is an integer. [5]

3 The equation of a curve is $\ln(x+y) = 3x^2y$.

Find the gradient of the curve at the point $(1, 0)$. [4]

4 **(a)** Show that $\sec^4\theta - \tan^4\theta \equiv 1 + 2\tan^2\theta$. [3]

(b) Hence or otherwise solve the equation $\sec^4 2\alpha - \tan^4 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$ for $0° < \alpha < 180°$. [5]

5 (a) By sketching a suitable pair of graphs, show that the equation $2 + e^{-0.2x} = \ln(1 + x)$ has only one root. [2]

(b) Show by calculation that this root lies between 7 and 9. [2]

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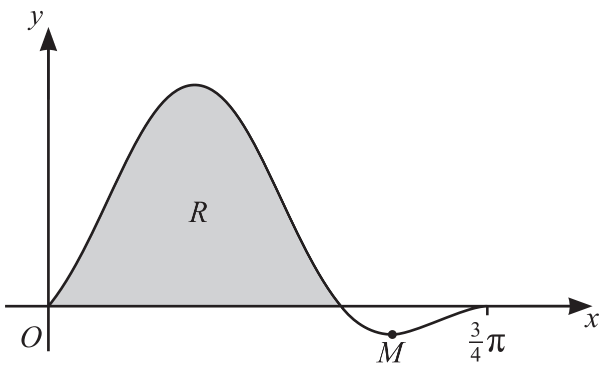
(c) Use the iterative formula

$$x_{n+1} = \exp\left(2 + e^{-0.2x_n}\right) - 1$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[exp(x) is an alternative notation for e^x.] [3]

6



The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

(a) Given that the x-coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M. [4]

(b) Find the exact area of the region R. [4]

7 Let $f(x) = \dfrac{5x^2 + 8x + 5}{(1+2x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the coefficient of x^3 in the expansion of $\mathrm{f}(x)$. [4]

8 **(a)** Given that $z = 1 + y\mathrm{i}$ and that y is a real number, express $\frac{1}{z}$ in the form $a + b\mathrm{i}$, where a and b are functions of y. [2]

(b) Show that $\left(a - \frac{1}{2}\right)^2 + b^2 = \frac{1}{4}$, where a and b are the functions of y found in part **(a)**. [3]

(c) On a single Argand diagram, sketch the loci given by the equations $\text{Re}(z) = 1$ and $\left|z - \frac{1}{2}\right| = \frac{1}{2}$, where z is a complex number. [3]

(d) The complex number z is such that $\text{Re}(z) = 1$. Use your answer to part **(b)** to give a geometrical description of the locus of $\frac{1}{z}$. [1]

9 The position vector of point A relative to the origin O is $\overrightarrow{OA} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

(a) State a vector equation for l. [2]

(b) The position vector of point B relative to the origin O is $\overrightarrow{OB} = -t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$, where t is a constant. The line l also passes through B.

Find the value of t. [3]

(c) The line m has vector equation $\mathbf{r} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} - \mathbf{j} + 3\mathbf{k})$. The acute angle between the directions of l and m is θ, where $\cos\theta = \frac{1}{\sqrt{6}}$.

Find the possible values of a. [5]

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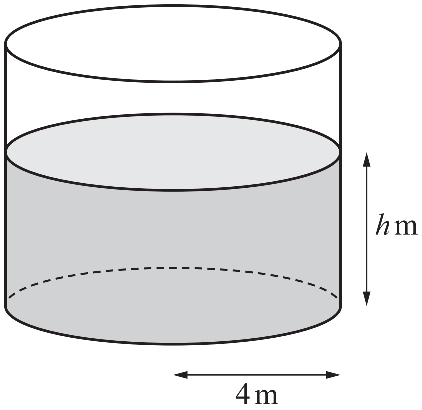

A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank.

(a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

(b) At time $t = 0$ the tap is opened. It is given that $h = 4$ when $t = 0$ and that $h = 2.25$ when $t = 20$.

Solve the differential equation to obtain an expression for t in terms of h, and hence find the time taken to empty the tank. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.