Cambridge Assessment International Education

# Cambridge International AS & A Level

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| CANDIDATE NAME | | | | | | | | | |
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*1939478240*

**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

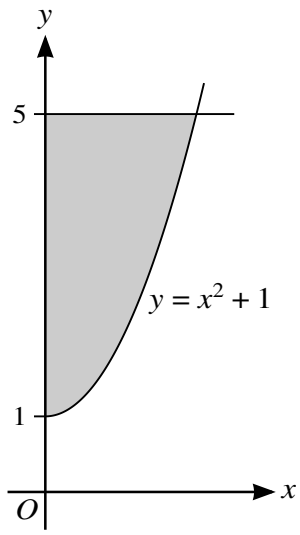
**1** The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

**2** The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

**3**




The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the $y$-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

**4** A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at $P$ the $y$-coordinate is increasing at 4 units per second and the $x$-coordinate is increasing at 6 units per second.

Find the $x$-coordinate of $P$. [4]

**5** Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

**6** The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

**(a)** Find the possible values of the constant $a$. [3]

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**(b)** Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

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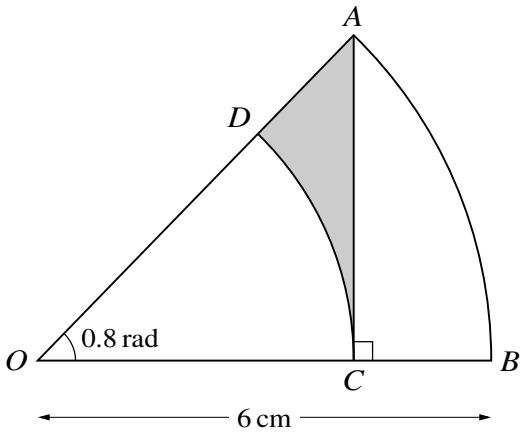


The diagram shows a sector $AOB$ which is part of a circle with centre $O$ and radius 6 cm and with angle $AOB = 0.8$ radians. The point $C$ on $OB$ is such that $AC$ is perpendicular to $OB$. The arc $CD$ is part of a circle with centre $O$, where $D$ lies on $OA$.

Find the area of the shaded region. [6]

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**8** A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

**(a)** For her first year, express her bonus as a percentage of her basic salary. [1]

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At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

**(b)** Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

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**9** **(a)** Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where $a$ and $b$ are constants. [2]

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The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

**(b)** Find an expression for $f^{-1}(x)$ and state the domain of $f^{-1}$. [3]

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The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

**(c)** For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

**(d)** State the largest value of $k$ possible for the composition fg to be defined. [1]

**10** The gradient of a curve at the point $(x, y)$ is given by $\frac{dy}{dx} = 2(x + 3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where $a$ is a positive constant.

**(a)** Find the value of $a$. [3]

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**(b)** Determine the nature of the stationary point. [3]

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**(c)** Find the equation of the curve. [4]

**11** **(a)** Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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**(b)** Find the set of values of $k$ for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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**(c)** For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of $k$ for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

**12** A diameter of a circle $C_1$ has end-points at $(-3, -5)$ and $(7, 3)$.

**(a)** Find an equation of the circle $C_1$. [3]

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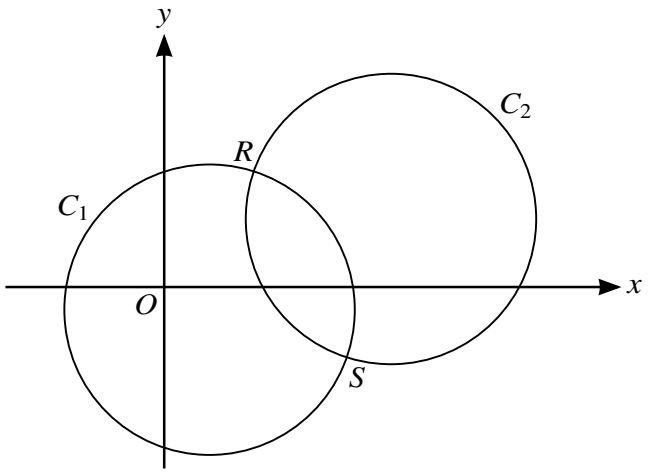


The circle $C_1$ is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle $C_2$, as shown in the diagram.

**(b)** Find an equation of the circle $C_2$. [2]

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The two circles intersect at points $R$ and $S$.

**(c)** Show that the equation of the line $RS$ is $y = -2x + 13$. [4]

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**(d)** Hence show that the $x$-coordinates of $R$ and $S$ satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

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## Additional Page

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