Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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CAMBRIDGE International Examinations

1 The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

2 The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

3 A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

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(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

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4 The function f is such that $f(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $f\left(\frac{1}{3}\pi\right) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

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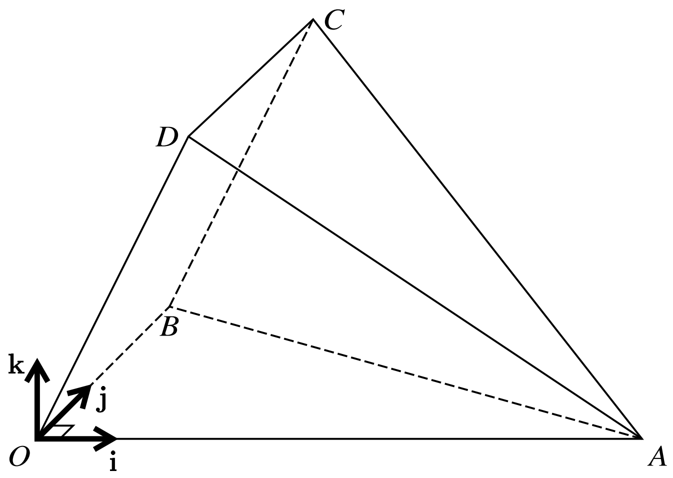

The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors **i** and **j** are parallel to OA and OB respectively and the unit vector **k** is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of **i**, **j** and **k**. [2]

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(ii) Use a scalar product to find angle *CAD*. [4]

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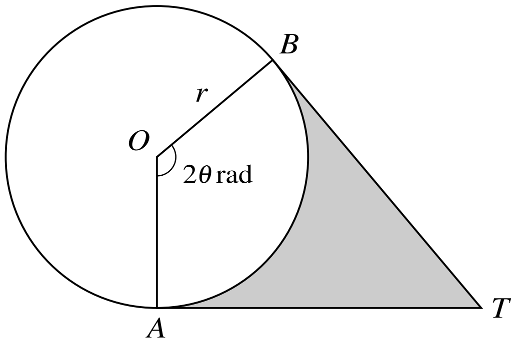

The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

7 The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

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(iv) For this value of k, find $g^{-1}(x)$. [3]

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8 Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

9 A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and (2, 5) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

10 **(i)** Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

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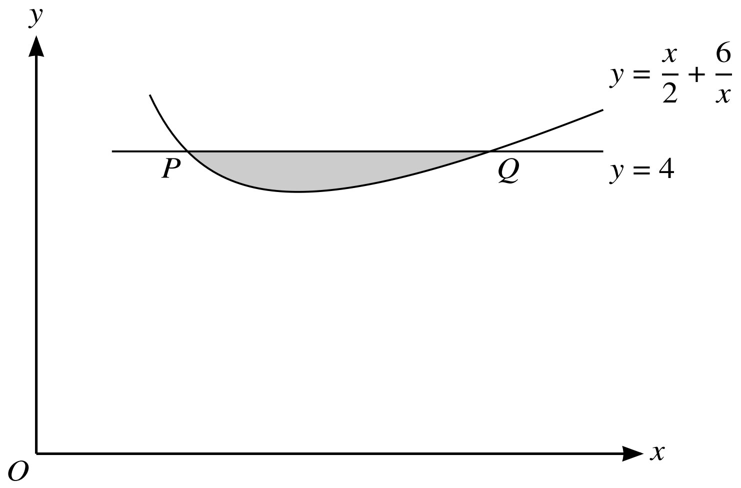

The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.