Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

8279021580

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

BLANK PAGE

1 The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

4 The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

5 Functions f and g are defined by

$$\mathrm{f}(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

.....

.....

.....

.....

.....

(b) Find the values of x for which $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the equation of the curve. [4]

.....

.....

.....

.....

.....

.....

.....

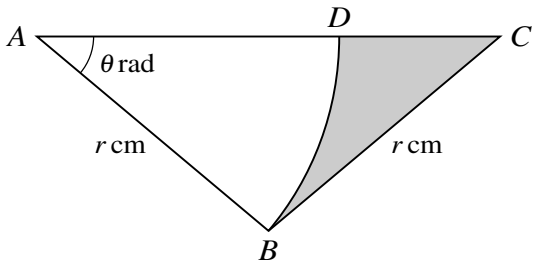
.....

.....

.....

.....

8



In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

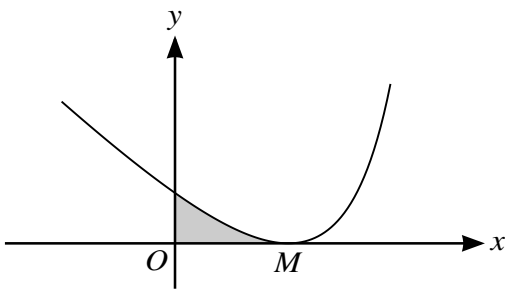
.....

.....

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

10



The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and $\int y\,\mathrm{d}x$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find, by calculation, the x-coordinate of M. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

.....

.....

.....

.....

(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

.....

.....

(ii) $k = 1$ [1]

.....

.....

(iii) $k = 3$ [1]

.....

.....

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$\begin{aligned} \mathrm{f}(x) &= 3\cos 2x + 2, \\ \mathrm{g}(x) &= \mathrm{f}(2x) + 4, \\ \mathrm{h}(x) &= 2\mathrm{f}\left(x + \tfrac{1}{2}\pi\right). \end{aligned}$$

(d) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(e) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ on to $y = \mathrm{h}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

BLANK PAGE

BLANK PAGE