Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1-\frac{1}{x}+\frac{1}{4x^2}$	**B1**	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
1(b)	$1+12x+60x^2+160x^3$	**B2, 1, 0**	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
1(c)	$their(1\times 12)+their(-1\times 60)+their(\frac{1}{4}\times 160)$	**M1**	Attempts at least 2 products where each product contains one term from each expansion.
	$[12-60+40=]\ -8$	**A1**	Allow $-8x$.
		2	

Question	Answer	Marks	Guidance
2	$kx^2+2x-k=kx-2$ leading to $kx^2+(-k+2)x-k+2\ [=0]$	*M1	Eliminate y and form 3-term quadratic. Allow 1 error.
	$(-k+2)^2-4k(-k+2)$	DM1	Apply b^2-4ac ; allow 1 error but a, b and c must be correct for *their* quadratic.
	$5k^2-12k+4$ or $(-k+2)(-k+2-4k)$	A1	May be shown in quadratic formula.
	$(-k+2)(-5k+2)$	DM1	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
	$\frac{2}{5}<k<2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
		5	

Question	Answer	Marks	Guidance
3	$3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)\ [=0]$	M1	Or similar partial factorisation; condone sign errors.
	$(2\tan\theta-1)\ (3\cos\theta+2)\ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	M1	OE. At least 2 out of 4 products correct.
	26.6º, 131.8º	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
	Alternative method for question 3		
	$6\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\frac{\sin\theta}{\cos\theta}\right)-2\ [=0]$ $6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta\ [=0]$ $2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)\ \ [=0]$	M1	Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors.
	$(2\sin\theta-\cos\theta)(3\cos\theta+2)\ \ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	M1	At least 2 out of 4 products correct.
	26.6º, 131.8º	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
		4	

Question	Answer	Marks	Guidance
4(a)	$\frac{5a}{1-\left(\pm\frac{1}{4}\right)}$	**B1**	Use of correct formula for sum to infinity.
	$\frac{8}{2}\left[2a+7(-4)\right]$	***M1**	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a=8a-112$ leading to $a=[28]$	**DM1**	Solve equation to reach a value of a.
	$a=28$	**A1**	Correct value.
		4	
4(b)	*their* $28+(k-1)(-4)=0$	**M1**	Use of correct method with *their* a.
	$[k=]8$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$a=5$	**B1**	
	$b=2$	**B1**	
	$c=3$	**B1**	
		3	

Question	Answer	Marks	Guidance
5(b)(i)	3	**B1**	
		1	
5(b)(ii)	2	**B1**	
		1	

Question	Answer	Marks	Guidance
6(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times their\ \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times their\ \frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	**B1**	
	One arc $\frac{1}{6}\times 12\pi$ leading to two arcs $2\times\frac{1}{6}\times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
		3	

Question	Answer	Marks	Guidance
6(b)	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)+6\pi\ \left[=6\pi-9\sqrt{3}+6\pi\right]$	M1 A1	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times 6^2\times their\left(\frac{\pi}{3}\right)\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)$	M1 A1	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\right)+$ $\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\ \left[=12\pi-18\sqrt{3}+9\sqrt{3}\right]$	M1 A1	M1 for attempt at strategy with values substituted: **2 ×segment + triangle** A1 if correct (unsimplified).
	Area $\left[=6\pi-9\sqrt{3}+6\pi\right]=12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
		4	

Question	Answer	Marks	Guidance
7(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2=13$ or $r=\sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
7(b)	$(x-5)^2+\ (5x-10-2)^2=13$	**M1**	Substitute $y=5x-10$ into *their* equation.
	$26x^2-130x+156\ [=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ [=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$	**M1**	Substitute $x=\frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25}-\frac{26y}{5}$ [= 0]	**A1 FT**	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)$ [= 0]	**M1**	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.
		7	

Question	Answer	Marks	Guidance
8(a)	$\{-3(x-2)^2\}$ $\{+14\}$	**B1 B1**	B1 for each correct term; condone a = 2, b = 14.
		2	
8(b)	[k =] 2	**B1**	Allow [x] $\leqslant$ 2.
		1	

Question	Answer	Marks	Guidance
8(c)	[Range is] [y] $\leqslant -13$	**B1**	Allow [f(x)] $\leqslant -13$, [f] $\leqslant -13$ but NOT $x \leqslant -13$.
		1	
8(d)	$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	**M1**	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	**A1**	Allow $\frac{y-14}{-3}$.
	$[\mathrm{f}^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>
	Alternative method for question 8(d)		
	$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	**M1**	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$= 2(\pm)\sqrt{\frac{14-x}{3}}$	**A1**	Allow $\frac{x-14}{-3}$.
	$[\mathrm{f}^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>
		3	

Question	Answer	Marks	Guidance
8(e)	[g(x) =] $\left\{-3(x+3-2)^2\right\}+\{14+1\}$	B2, 1, 0	OR $\left\{-3(x+3)^2\right\}+\{12(x+3)\}+\{3\}$
	$g(x)=-3x^2-6x+12$	B1	
		3	

Question	Answer	Marks	Guidance
9(a)	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}\ [+c]$	B2, 1, 0	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$	M1	Substitute f(1) = $-\frac{1}{3}$ into an integrated expression and evaluate c.
	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	A1	OE.
		4	

Question	Answer	Marks	Guidance
9(b)	$2x^4 - 7x^2 - 4\ [=0]$	**M1**	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4[=0]$.
	$(2x^2+1)(x^2-4)\ [=0]$	**M1**	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. y. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x = [\pm]2$	**A1**	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \quad \text{leading to}\right]\left(2, -\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \quad \text{leading to}\right]\ \left(-2, \frac{26}{3}\right)$	**B1 B1**	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
9(c)	$f''(x) = 4x + 8x^{-3}$	**B1**	OE
		1	

Question	Answer	Marks	Guidance
9(d)	$f''(2) = 9 > 0$ MINIMUM at $x = \textit{their}\ 2$	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = 2$.
	$f''(-2) = -9 < 0$ MAXIMUM at $x = \textit{their} - 2$	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = -2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2) > 0$ MIN and $f''(-2) < 0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	M1	FT on *their* $x = [\pm]2$
	MINIMUM at $x = \textit{their}\ 2$, MAXIMUM at $x = \textit{their}\ 2$	A1 FT	FT on *their* $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ $-/0/+$ MIN and $f'(-2)$ $+/0/-$ MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	B1 B1	Need correct coordinates in **(b)** for this method.
		2	

Question	Answer	Marks	Guidance
10(a)	$\left\{\dfrac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	**B2, 1, 0**	Attempt to integrate
	$-\frac{2}{3}[0-1]$	**M1**	M1 for applying limits $1 \to \infty$ to an integrated expression (either correct power or dividing by their power).
	$\dfrac{2}{3}$	**A1**	
		4	
10(b)	$[\pi]\int y^2 \mathrm{d}x = [\pi]\int (3x-2)^{-3}\,\mathrm{d}x = [\pi]\dfrac{(3x-2)^{-2}}{-2\times 3}$	***M1 A1**	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\dfrac{1}{6}\right]\left[\dfrac{1}{16}-1\right]$	**DM1**	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\dfrac{5\pi}{32}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
10(c)	$\frac{dy}{dx} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$	**M1**	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$	***M1**	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y - 1 = \frac{2}{9}(x-1)$ OR evaluates c	**DM1**	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{\textit{their}\ \frac{dy}{dx}}$.
	At A, $y = \frac{7}{9}$	**A1**	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	