wjec cbac

GCSE MARKING SCHEME

SUMMER 2023

GCSE MATHEMATICS UNIT 2 – HIGHER TIER 3300U60-1

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INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCSE MATHEMATICS

SUMMER 2023 MARK SCHEME

Unit 2: Higher Tier	Mark	Comments
1.(a)		FT until 2 nd error.
7 + 5x - 10 = 3x + 8 or equivalent.	B1	Bracket must be expanded or correct division by 5 e.g $x - 2 = \frac{3x}{5} + \frac{1}{5}$ (but not $x - 2 = \frac{3x + 1}{5}$)
2x = 11 OR $-11 = -2x$	B1	Or equivalent Correctly simplifying the equation to a single <i>x</i> term and number term (e.g. $2x - 11 = 0$).
$x = \frac{11}{2}$ or 5.5 or equivalent.	B1	Mark final answer. Correct answer implies B1B1B1. Do not allow $-x = \frac{-11}{2}$ or $x = \frac{-11}{-2}$ A final answer of '11 ÷ 2' is B1B1B0. If FT leads to a whole number answer, it must be
		shown as a whole number. Otherwise, accept a fraction. Allow any decimal answer to be rounded or truncated to 1 or more decimal place.
		Allow B1B1B1 for a correct embedded answer BUT only B1B1B0 if contradicted by $x \neq \frac{11}{2}$ or equivalent.
		Note: 12x - 24 = 3x + 8 B0 9x = 32 B1 (FT) $x = \frac{32}{9}$ or $3 \cdot 5(55)$ or $3 \cdot 6$. B1 (FT)
		If no marks awarded, award SC1 for sight of one of the following: • $5x - 10$ • $12x - 24$.
1.(b) $2f = 13 - h$ or $h - 13 = -2f$	B1	Or equivalent.
$f = \frac{13 - h}{2} \text{ or } \frac{h - 13}{-2} = f$ or equivalent	B1	Or equivalent. Must not come from incorrect working. Mark final answer. FT only from $\pm 2f = \pm 13 \pm h$. Unsupported $f = \pm 13 \pm h$ implies B0B1 unless B2. ± 2 Award B1B0 for $-f = h - 13$ or equivalent. If no marks, award SC1 for a final answer of either: • $f = (13 - h) \div 2$ with or without brackets • $f = (h - 13) \div -2$ with or without brackets • $\frac{13 - h}{2}$ ('f =' missing). • $\frac{h - 13}{-2}$ ('f =' missing).
1.(c) $5(3x-7y)$	B1	Mark final answer. Allow $-5(-3x + 7y)$ or $5(3x + -7y)$.

2.(a) P(Bronze) = 0.2 AND P (No Prize) = 0.6	B2	The values in the table takes precedence
2.(a) $P(Bronze) = 0.2$ AND $P(No Prize) = 0.6$ or equivalent	B2	The values in the table takes precedence. Award B1 for one of the following:
		• $P(Bronze) = 0.2$ (must be clearly identified)
		• $P(No Prize) = 0.6$
		• $P(Bronze) + P(No Prize) = 0.8$
		• $P(Bronze) = \frac{1}{3} P(No Prize) provided both <1.$
2.(b) $15 \div 0.02 \times 0.18$ or	M1	Must be for a complete method e.g.
15 × 9 or equivalent		• $15 \div 2 = 7.5$ $7.5 \times 18 = 135$
		• 750 - (450 + 150 + 15)
		• 0.02 : 0.18 15 : 125 (2 : 0.18 :: 750, cr.15 :: 0)
= 135	A1	15 : 135 (e.g 0⋅18 × 750, or 15 × 9)
		Award M1 A1 for a final answer of 15 : 135.
		Sight of 135 as a numerator in a fraction < 1 implies M1A0.
3.		Correct evaluation regarded as enough to identify if
		negative or positive.
		If evaluations not seen accept 'too high' or 'too low'. Look out for equating $x^3 - 8x = -3$
		3
One correct evaluation $2 \le x \le 3$	B1	$\frac{x}{2} = \frac{x^3 - 8x + 3}{-5}$
2 correct evaluations $2.55 \le x \le 2.75$,	B1	2·1 -4·539 2·55 -0·818
(one evaluation < 0, one evaluation > 0)		$2 \cdot 2 = -3 \cdot 952 = 2 \cdot 61 = -0 \cdot 1004$
		2·3 -3·233 2·62 0·0247
2 correct evaluations $2.55 \le x \le 2.65$,	M1	2·4 -2·376 2.63 0·1514
(one evaluation < 0, one evaluation > 0)		2·5 −1·375 2.64 0·2797
	_	2.6 -0.224 2.65 0.409
$x = 2 \cdot 6$	A1	2.7 1.083 2.75 1.796 2.8 2.552
		2·8 2·552 2·9 4·189
		3 6
		-
		Unsupported $x = 2.6$ is awarded B0B0M0A0.
		An answer of $x = 2.6$ can only be awarded M1A1,
		following sight of 2 correct evaluations
		$2.55 \le x \le 2.65$
		(one evaluation < 0 , one evaluation > 0).
4.(a) 1·2	B2	Mark final answer.
		Award B1 for one of the following:
		• sight of 1·1(5519).
		 an answer of 1.20.
		Do not oward D2 or D4 for anowars obtained from
		Do not award B2 or B1 for answers obtained from incorrect work (e.g. rounding and/or estimating).
4.(b) 0.043	B2	Mark final answer.
		Award B1 for sight of one of the following:
		• 1
		23
		• 1÷23
		 0.0434() 0.0435
		 0.0435 0.04.
		 ● 0.04.
4.(c)(i) 12	B1	
4.(c)(ii) 5	B1	

 5. Method to eliminate one variable e.g. equal coefficients AND appropriate intention to add or subtract or use a method of substitution. 	M1	Allow one error in one term (not the term with equal coefficients).
First variable found $x = 4.3$ or $y = 2.6$ or equivalent	A1	CAO Award A0 for expressing the final answers in a form such as $y = \frac{33 \cdot 8}{13}$.
Substitute to find the 2 nd variable.	m1	FT substitution of their '1 st variable' if M1 gained.
Second variable found	A1	No marks for 'trial and improvement'. No marks for an unsupported answer.
6.(a) (x =) $\sin^{-1} \frac{7 \cdot 7}{11 \cdot 3}$ or $\sin^{-1} \frac{7 \cdot 7 \times \sin 90}{11 \cdot 3}$ or equivalent	M2	Check diagram for answers Award M1 for one of the following: • $\sin x = \frac{7 \cdot 7}{11 \cdot 3} = 0.68(1)$ • $\frac{\sin x = \sin 90}{11 \cdot 3}$ or equivalent 7.7 11.3
Allow an answer between 42.8 and 43(°) ISW	A1	Allow correct angles given in radians or gradians:MethodRadiansGradians $\sin^{-1} \frac{7 \cdot 7}{11 \cdot 3}$ $0 \cdot 7496$ $47 \cdot 727$ $\sin^{-1} \frac{7 \cdot 7 \times \sin 90}{11 \cdot 3}$ $0 \cdot 655$ $47 \cdot 001$
6.(a) <u>Alternative method</u> Correct use of a 'two-step' method.	M2	A partial trigonometric method is M0.
Allow an answer between 42.8 and 43(°) ISW	A1	Allow 42⋅8(°) Allow correct angles given in radians or gradians.

$BED = 43(^{\circ})$ provided 'their $x' \neq 45^{\circ}$. Note: DBE must be acute for B1. May be implied in further work.Valid method to find the length DE M2 $DE = 13 \cdot 1 \times \tan 47$ M2 $DE = \underline{13 \cdot 1} \times \tan 43$ M2 $DE = \underline{13 \cdot 1} \times \sin 43$ M2 $DE = \underline{13 \cdot 1} \times \sin 43$ <	BED = 43(°) alid method to find the length DE an 47 $DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$		Strict FT for $DBE = 90 - $ 'their x' or $BED = $ 'their x', provided 'their x' $\neq 45^{\circ}$. Note: DBE must be acute for B1. May be implied in further work. If B1 already awarded for 'their angle DBE but then 'their angle BED ' is incorrect and 'their BED ' is then used (or vice versa) for either M2 or M1, then award B0 previously.
$BED = 43(^{\circ})$ provided 'their $x' \neq 45^{\circ}$. Note: DBE must be acute for B1. May be implied in further work.Valid method to find the length DE M2 $DE = 13 \cdot 1 \times \tan 47$ M2 $DE = \frac{13 \cdot 1}{\tan 43}$ M2 $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ M2 $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ Or award M2 for correct use of a 'two-step' method (e.g. 'Pythagoras and similar triangles' or 'Pythagoras and correct trigonometric relationship').FT 'their angle DBE or 'their angle BED ' provided not 0°, 45°, 90° or 180°.	lid method to find the length <i>DE</i> an 47 $DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$	M2	provided 'their $x' \neq 45^{\circ}$. Note: <i>DBE</i> must be acute for B1. May be implied in further work. If B1 already awarded for 'their angle <i>DBE</i> ' but then 'their angle <i>BED</i> ' is incorrect and 'their <i>BED</i> ' is then used (or vice versa) for either M2 or M1, then award B0 previously.
Valid method to find the length DE M2May be implied in further work. $DE = 13 \cdot 1 \times \tan 47$ M2If B1 already awarded for 'their angle DBE but the 'their angle BED ' is incorrect and 'their BED ' is the used (or vice versa) for either M2 or M1, then awa B0 previously. $DE = \frac{13 \cdot 1}{\tan 43}$ Or award M2 for correct use of a 'two-step' method (e.g. 'Pythagoras and similar triangles' or 'Pythagorad and correct trigonometric relationship').FT 'their angle DBE or 'their angle BED' provided not 0°, 45°, 90° or 180°.	$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$	M2	May be implied in further work. If B1 already awarded for 'their angle <i>DBE</i> ' but then 'their angle <i>BED</i> ' is incorrect and 'their <i>BED</i> ' is then used (or vice versa) for either M2 or M1, then award B0 previously.
Valid method to find the length DE M2If B1 already awarded for 'their angle DBE but the 'their angle BED ' is incorrect and 'their BED ' is the used (or vice versa) for either M2 or M1, then awa B0 previously. $DE = \underline{13.1}$ tan 43 $DE = \underline{13.1 \times \sin 47}$ sin 43Or award M2 for correct use of a 'two-step' method (e.g. 'Pythagoras and similar triangles' or 'Pythagora and correct trigonometric relationship').FT their angle DBE or 'their angle BED ' provided not 0°, 45°, 90° or 180°.	$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$	M2	If B1 already awarded for 'their angle <i>DBE</i> ' but then 'their angle <i>BED</i> ' is incorrect and 'their <i>BED</i> ' is then used (or vice versa) for either M2 or M1, then award B0 previously.
$DE = 13.1 \times \tan 47$ $DE = \frac{13.1}{\tan 43}$ $DE = \frac{13.1 \times \sin 47}{\sin 43}$ $E = 13.1 \times $	$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$	M2	'their angle <i>BED</i> ' is incorrect and 'their <i>BED</i> ' is then used (or vice versa) for either M2 or M1, then award B0 previously.
$DE = 13.1 \times \tan 47$ $DE = \frac{13.1}{\tan 43}$ $DE = \frac{13.1 \times \sin 47}{\sin 43}$ $DE = 13.1 \times \sin $	$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$		used (or vice versa) for either M2 or M1, then award B0 previously.
$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = 13 \cdot$	$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{2}$		B0 previously.
$DE = \frac{13 \cdot 1}{\tan 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E = \frac{13 \cdot 1 \times \sin 47}{\sin 47}$ $E =$	tan 43 <i>DE</i> = <u>13⋅1 × sin 47</u>		
$DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ (e.g. 'Pythagoras and similar triangles' or 'Pythagoras and correct trigonometric relationship'). FT 'their angle <i>DBE</i> or 'their angle <i>BED</i> ' provided not 0°, 45°, 90° or 180°.	$DE = \frac{13 \cdot 1 \times \sin 47}{2}$		
$DE = \frac{13 \cdot 1 \times \sin 47}{\sin 43}$ and correct trigonometric relationship'). FT 'their angle <i>DBE</i> ' or 'their angle <i>BED</i> ' provided not 0°, 45°, 90° or 180°.			
sin 43 FT 'their angle <i>DBE</i> ' or 'their angle <i>BED</i> ' provided not 0°, 45°, 90° or 180°.			
not 0°, 45°, 90° or 180°.			
			FT 'their angle <i>DBE</i> ' or 'their angle <i>BED</i> ' provided
Award M1 for one of the following			not 0°, 45°, 90° or 180°.
			Award M1 for one of the following:
• tan 47 = <u>DE</u>			
13.1			-
• $\tan 43 = \frac{13 \cdot 1}{DF}$			
• $\underline{DE} = \underline{13.1}$ or equivalent			
sin 47 sin 43			
For all M2 or M1 scenarios, ET their clearly stated			For all M2 or M1 scenarios, FT their clearly stated or
shown angle <i>BED</i> or <i>DBE</i> where appropriate.			
For 13.1 x sin 47. FT their clearly stated or shown			For $13.1 \times sin 47$ FT their clearly stated or shown
sin 43			
angles <i>BED</i> and <i>DBE</i> <u>only</u> if <i>BED</i> + <i>DBE</i> = 90°.			angles <i>BED</i> and <i>DBE</i> <u>only</u> if <i>BED</i> + <i>DBE</i> = 90°.
DE in the range 14.04 to 14.1 (cm) ISW A1 Allow 14 from correct workings.	n the range 14.04 to 14.1 (cm) ISW	A1	Allow 14 from correct workings. FT from M2 only and provided that angle is acute and
leads to a positive answer.			
Award B1M2A0 for any of the following unsupport answers:			Award B1M2A0 for any of the following unsupported
Method Radians Gradians			
13·1 × tan 47 -1·63 to 1 11·92 to 12			13.1 × tan 47 -1.63 to 1 11.92 to 12
<u>13.1</u> -8.743 to 16.35 to 16.1			
tan 43 -5-36			tan 43 -5·36
$\frac{13 \cdot 1 \times \sin 47}{13 \cdot 1 \times \sin 47}$ = 1.95 to 1.08 = 14.1 to 14.2			$13.1 \times \sin 47$ 1.05 to 1.09 14.1 to 14.24
sin 43 -1.95 to 1.08 14.1 to 14.2			
		1	

7.(a) $\times 0.95^4$	B1	
7.(b) Sight of 0.83 OR 83%	B1	Allow (100 –17 =) 83
<u>3569</u> or <u>3569</u> × 100 or equivalent 0⋅83 83	M1	FT 'their 1 – 0.17 ' provided <1 or 'their 100% – 17%' provided < 100%.
= 4300	A1	Award B1M1A1 for an embedded answer (e.g. $0.83 \times 4300 = 3569$ or $\frac{3569}{4300} \times 100 = 83$), BUT only B1M1A0 if contradicted by stating original amount $\neq 4300$.
		Unsupported 4300 is awarded B1M1A1.
8. $\frac{\pi \times r^2}{2} = 77$ or equivalent	M1	Check diagram for answers.
$r^2 = 49(.0) \text{ or } r^2 = \frac{154}{\pi}$	m1	Sight of $49(0)$ implies M1m1.
<i>r</i> = 7(·0…)	A1	FT 'their r^{2} ' provided M1 awarded. 7 must not be from incorrect working.
(Area of trapezium =) $\frac{2 \times 7(\cdot 0) + 22}{2} \times 7(\cdot 0)$ or equivalent	M1	FT 'their derived or stated r '.
= 126·0(…)(cm²)	A1	Accept 126.1 or 126 (cm ²) Mark final answer.
9.	B2	B1 for 2 correct vertices within a triangle, e.g. A and 1 other vertex OR for a triangle of correct shape, size and orientation in incorrect position OR consistent correct use of an incorrect negative scale factor OR for 3 correct vertices (A implied) in the correct location not joined to form the triangle.

		1
10.(a) (Reflex angle D =) 180 + 360/5 OR 360 – 3×180/5	M1	Or any other complete method. May be seen in stages.
= 252(°)	A1	Award for sight of 252(°) if not contradicted by further incorrect work.
(Arc length CE =) $252/360 \times \pi \times 2 \times 11$	M1	FT for M1 for 'their derived angle'.
48·3() OR 48·4 OR 77π/5 OR 15 ² / ₅ π OR 15·4π(cm) o.e.	A1	ISW Accept 48(cm) from correct working. 12089/250 from using 3·14 Allow a FT for this mark provided the angle used is >180°. If no marks award SC1 for sight of 108(°) OR 72(°)
Organisation and Communication	OC1	 For OC1, candidates will be expected to: present their response in a structured way explain to the reader what they are doing at each step of their response lay out their explanation and working in a way that is clear and logical write a conclusion that draws together their results and explains what their answer means
Accuracy of writing	W1	 For W1, candidates will be expected to: show all their working make few, if any, errors in spelling, punctuation and grammar use correct mathematical form in their working use appropriate terminology, units, etc.
10.(b) $\left(\frac{671}{11}\right)^2$ (= 450241/121) OR 61 ²	M1	
= 3721	A1	Answer in the sentence takes precedence. If no marks, allow SC1 for 3721/25 OR $148\frac{21}{25}$ OR 148.84 [from (671/55) ²].
11. $x^{2}(a+1) = b \text{ OR } -x^{2}(a+1) = -b \text{ OR}$ $x^{2}(-a-1) = -b$	B1	FT until 2^{nd} error. x^2 or $-x^2$ factorised.
$x^2 = \frac{b}{a+1} \operatorname{OR} - x^2 = \frac{-b}{-a-1}$	B1	Isolating the x^2 or $-x^2$. Allow a FT from $2x^2 = \frac{b}{a}$.
$x = \pm \sqrt{\frac{b}{a+1}}$	B1	B0 for $\sqrt{[b \div (a + 1)]}$ (use of the division sign). Allow omission of \pm . Mark final answer.
		Note: $2x^2 = \frac{b}{a}$ B0
		$x^2 = \frac{b}{2a}$ B1 (FT)
		$x = (\pm) \sqrt{\frac{b}{2a}}$ B1 (FT)

12.(a) $2(2x+3)(2x-3)$	B3	Award B3-1 for a correct answer followed by further incorrect work. Award B2 for the sight of any one of the following: • $(4x + 6)(2x - 3)$ • $(4x - 6)(2x + 3)$ • $8(x + 3/2)(x - 3/2)$ • $(2x + 3)(2x - 3)$ • $2(2x + 3)(2x - 3)$ • $2(2x - 3)(2x - 3)$ Award B1 for the sight of any one of the following: • $2(4x^2 - 9)$ • $8(x^2 - 9/4)$ • $(4x + 6)(2x + 3)$ • $(4x - 6)(2x - 3)$ • $(x + 3/2)(x - 3/2)$ If no marks: Allow SC2 for $(2\sqrt{2}x + 3\sqrt{2})(2\sqrt{2}x - 3\sqrt{2})$ o.e. OR other valid, equivalent 'factorisation', e.g. (8x - 12)(x + 1.5) o.e. Allow SC1 for $(\sqrt{8}x + \sqrt{18})(\sqrt{8}x - \sqrt{18})$ o.e.
12.(b) 3/2 AND -3/2	B1	Or equivalent for either roots. FT if possible, provided exactly 2 possible distinct solutions.
12.(c) A <u>positive</u> quadratic curve passing through (0,-18) as a minimum with -18 indicated on the y-axis AND passing through (–3/2,0) and (3/2,0) which are indicated on the x-axis.	B2	 FT for x-axis intersections, provided exactly 2 possible distinct solutions. Award B1 for any one of the following: A positive quadratic curve passing through (0, -18) as a minimum with -18 indicated on the y-axis OR A quadratic curve (either positive or negative) passing through (-3/2,0) and (3/2,0) which are indicated on the x-axis. If the conditions for B2 are met, then only allow B1 for concave and/or convex curvature above the <i>x</i>-axis.

13.(a) $\frac{2}{4} \times \frac{3}{5} \times \frac{3}{6}$	M1	
$= \frac{13}{120} \text{ ISW} (= \frac{3}{20} \text{ or } 0.15) \text{ oe}$	A1	If no marks, award SC1 for: $\frac{a}{4} \times \frac{b}{5} \times \frac{c}{6}$ correctly evaluated with at least two of a, b and c correct OR an answer of $\frac{48}{120}$ (= $\frac{2}{5}$ or 0.4 or equivalent) from assuming that 1 is prime, $\frac{3}{4} \times \frac{4}{5} \times \frac{4}{6}$ OR an answer of $\frac{4}{120}$ (= $\frac{1}{30}$ or 0.03(3) or equivalent) from excluding 2 as prime, $\frac{1}{4} \times \frac{2}{5} \times \frac{2}{6}$
13.(b) $[1 - P(1, 1, 1) - 3 \times P(1, 1, 2) =]$		
$1 - 4 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}$ OR $\frac{120 - 4}{120}$	M2	Award M1 for sight of: • $4 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}$ or equivalent (from P(1, 1, 1) + 3×P(1, 1, 2)) OR • $1 - 2 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}$ or equivalent (from $1 - P(1, 1, 1) - P(1, 1, 2)$) OR • $1 - 3 \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}$ or equivalent (from $1 - 3 \times P(1, 1, 2)$) OR • $1 - \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6}$ or equivalent (from $1 - P(1, 1, 1)$), note: this may include further incorrect work, e.g. $\frac{117}{120} \left(=\frac{39}{40}\right)$ from $1 - \frac{1}{4} \times \frac{1}{5} \times \frac{1}{6} - \frac{1}{4} \times \frac{1}{5} \times \frac{2}{6}$ or equivalent OR • $120-4$ (120 permutations – the 4 permutations not allowed)
$=\frac{116}{120}$ ISW $(=\frac{29}{30})$ oe	A1	CAO Allow 0.96 OR 0.966(6) OR 0.967 oe. Allow 0.97 from correct working. If no marks award SC1 for an answer of $\frac{114}{120} \left(=\frac{19}{20}\right)$ from 1–3×P(1,1,1)–3×P(1,1,2) *Note: award M2 for a <u>fully correct</u> method of listing all the ways leading to totals greater than 4, but M0 for a partial attempt.
14. Sight of 22(cm) OR ³ √10648 (cm) √(22 ² +22 ² +22 ²) OR √(1452)	B1 M2	FT 'their 22' for M2. May be seen in stages. (If the answer is completed in stages, then any arithmetic errors in intermediate answers can be ignored, allowing for a FT for the M marks). M1 for 22 ² +22 ² +22 ² (=1452)
= $38 \cdot 1($ cm) OR $22\sqrt{3}$ (cm) OR a value that rounds to $38 \cdot 1$ (cm), e.g. $38 \cdot 09$ (cm)	A1	CAO. Allow 38(cm) from correct working <u>provided their</u> answer would round to 38.1.

15.		A correct common denominator may be shown throughout for first 3 marks. Also, look out for alternative, correct methods leading to the same quadratic equated to zero.
3x - 7 + x - 2 = (x - 2)(3x - 7) oe Sight of $3x^2 - 7x - 6x + 14$ $3x^2 - 17x + 23 = 0 \text{ OR } -3x^2 + 17x - 23 = 0$	B1 B1 B1	For multiplying throughout the numerator of all terms by the common denominator. Or equivalent. May be seen in the denominator. '= 0' required, but may be implied by an attempt to use the quadratic formula or if $a = 3, b = -17$, c = 23 used in the quadratic formula. FT from B1B0 from one error only.
$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \times 3 \times 23}}{2 \times 3}$	M1	This substitution into the formula must be seen for M1, otherwise award M0A0A0. FT 'their derived quadratic equation equated to 0', (but not $3x^2 - 13x + 14 = 0$), provided of equivalent difficulty (<i>a</i> , <i>b</i> and <i>c</i> must be non-zero). Allow one slip in substitution <u>for M1 only</u> , but must be correct formula.
$x = \frac{17 \pm \sqrt{13}}{6}$	A1	Can be implied from at least one correct value of x evaluated.
x = 3.43 with $x = 2.23$	A1	CAO for their quadratic equation. Answers must be given to 2 decimal places. On FT solutions must require rounding.
16. Initially calculating angle BDC		
$\cos BDC = \frac{5^2 + 7^2 - 11^2}{2 \times 5 \times 7} \qquad (= -47/70)$	M2	M1 for $11^2 = 5^2 + 7^2 - 2 \times 5 \times 7 \times \cos BDC$
(BDC =) 132(·17…°)	A1	The correct angles given in: Radians (2·3038 to 2·3073) or Gradians (146·6666 to 146·8888).
		Allow a trial and improvement method for M2A1 but length of side BC must be in the range 10.95(cm) to 11.05(cm).
		Sight of $28(\cdot 13^{\circ})$ OR $132(\cdot 17^{\circ})$ OR $19(\cdot 68^{\circ})$, calculated from <u>one</u> use of the rearranged cosine rule in triangle BCD and attributed to the wrong angle, stated or seen on the diagram, is M0 SC1.
sin BAD = <u>7×sin [180–132(·17…°)]</u> 13	M2	FT 'their stated or derived BDC'. M1 for $\frac{\sin BAD}{7} = \frac{\sin [180-132(\cdot 17^{\circ})]}{13}$ o.e.
(BAD =) Answers in the range 23.4(°) to 23.6(°)	A1	Mark final answer for angle BAD. Allow a correctly rounded answer to a whole number provided an answer in the range 23.4(°) to 23.6(°) is seen.

Alternative method 1		
16. <u>Initially calculating angle BCD</u>		
$\cos BCD = \frac{5^2 + 11^2 - 7^2}{2 \times 5 \times 11} \qquad (=97/110)$	М2	<i>M1</i> for $7^2 = 5^2 + 11^2 - 2 \times 5 \times 11 \times \cos BCD$
(BCD =) 28(·13…°)	A1	The correct angles given in: Radians (0·4886 to 0·4911) or Gradians (31·1111 to 31·2666).
		Allow a trial and improvement method for M2A1 but length of side BD must be in the range $6.95(cm)$ to $7.05(cm)$.
		Sight of 28(·13°) OR 132(·17°) OR 19(·68°), calculated from <u>one</u> use of the rearranged cosine rule in triangle BCD and attributed to the wrong angle, stated or seen on the diagram, is M0 SC1.
sin BAD = <u>11×sin 28(·13…°)</u> 13	М2	FT 'their stated or derived BCD'. M1 for $\underline{\sin BAD} = \underline{\sin 28(\cdot 13^{\circ})}$ o.e. 11 13
(BAD =) Answers in the range $23.4(^{\circ})$ to $23.6(^{\circ})$	A1	Mark final answer for angle BAD. Allow a correctly rounded answer to a whole number provided an answer in the range 23.4(°) to 23.6(°) is seen.
Alternative method 2		
16. <u>Multistep method to calculate angle BDC</u> (First BCD, then CBD or vice versa)		
$\cos BCD = \frac{5^2 + 11^2 - 7^2}{2 \times 5 \times 11} (=97/110) \text{ OR}$	М2	A complete, correct method must be seen for the first M2.
$\cos CBD = \frac{7^2 + 11^2 - 5^2}{2 \times 7 \times 11} $ (=145/154)		M1 for one of the following: $7^{2} = 5^{2} + 11^{2} - 2 \times 5 \times 11 \times \cos BCD OR$ $5^{2} = 7^{2} + 11^{2} - 2 \times 7 \times 11 \times \cos CBD$
(BCD =) 28(·13°) OR (CBD =) 19(·68°)	A1	The correct angles given in: Radians (BCD \approx 0·49 or CBD \approx 0·34) or Gradians (BCD \approx 31·2 or CBD \approx 21.9).
$sin CBD = \frac{5 \times sin [28(\cdot 13^{\circ})]}{7} OR$		
sin BCD = <u>7×sin 19(·68…°)</u> 5		
(CBD =) 19(·68°) OR (BCD =) 28(·13°)		
(BDC =) 180 – [28(·13°)+19(·68°)]		
= 132(·17…°)		
sin BAD = <u>7×sin [180–132(·17…°)]</u> 13	М2	This M2 (or M1 below) is for a complete multistep method, i.e. it includes all the previous steps. FT 'their stated or derived BDC'. M1 for $\underline{\sin BAD} = \underline{\sin [180 - 132(.17^{\circ})]}{7}$ o.e. $\frac{13}{7}$
$(BAD =)$ Answers in the range $23.4(^{\circ})$ to $23.6(^{\circ})$	A1	Mark final answer for angle BAD. Allow a correctly rounded answer to a whole number provided an answer in the range 23.4(°) to 23.6(°) is seen.

<u>Alternative method 3</u> 16. <u>Multistep method to calculate angle BDC</u> (First BCD, then BDC OR first CBD, then BDC)		Ambiguous case
$\cos BCD = \frac{5^2 + 11^2 - 7^2}{2 \times 5 \times 11}$ (=97/110) OR	М2	A complete, correct method must be seen for the first M2. M1 for one of the following:
$\cos CBD = \frac{7^2 + 11^2 - 5^2}{2 \times 7 \times 11} (=145/154)$		$7^{2} = 5^{2} + 11^{2} - 2 \times 5 \times 11 \times \cos BCD OR$ $5^{2} = 7^{2} + 11^{2} - 2 \times 7 \times 11 \times \cos CBD$
(BCD =) 28(·13°) OR (CBD =) 19(·68°)	A1	The correct angles given in: Radians (BCD \approx 0·49 or CBD \approx 0·34) Gradians (BCD \approx 31·2 or CBD \approx 21.9).
$sin BDC = \frac{11 \times sin [28(\cdot 13^{\circ})]}{7} (=47 \cdot 82^{\circ}) OR$		
$sin BDC = \frac{11 \times sin 19(.68^{\circ})}{5}$ (=47.82^{\circ})		
(BDC = 180–47(·82…°)) = 132(·17…°)		
sin BAD = <u>7x sin [180–132(·17…°)]</u> 13	М2	This M2 (or M1 below) is for a complete multistep method, i.e. it includes all the previous steps. FT 'their stated or derived BDC'. M1 for $\underline{\sin BAD} = \underline{\sin [180 - 132(\cdot 17^{\circ})]}{7}$ o.e. 7
(BAD =) Answers in the range $23.4(^{\circ})$ to $23.6(^{\circ})$	A1	Mark final answer for angle BAD. Allow a correctly rounded answer to a whole number provided an answer in the range 23.4(°) to 23.6(°) is seen.