

Cambridge IGCSE[™]

CANDIDATE NAME				
CENTRE NUMBER		CANDIDATE NUMBER		
CAMBRIDGE	06	07/51		
Paper 5 Investigation (Core)		October/November 2021		
		1 hour 10 m	inutes	
You must answer on the question paper.				
No additional m	natorials are needed			

No additional materials are needed.

INSTRUCTIONS

- Answer all questions. •
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs. •
- Write your name, centre number and candidate number in the boxes at the top of the page. •
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid. •
- Do not write on any bar codes. •
- You should use a graphic display calculator where appropriate. •
- You may use tracing paper. •
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods. •
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working • to communicate your mathematics clearly and precisely.

This document has 8 pages. Any blank pages are indicated.

INFORMATION

- The total mark for this paper is 36.
- The number of marks for each question or part question is shown in brackets [].

Answer **all** the questions.

ADDING SQUARE NUMBERS

This investigation looks at adding two or more square numbers to make another square number. In this investigation all numbers are positive integers.

1 Complete the list of the first six square numbers.

 $1^{2} = 1$ $2^{2} = \dots$ $3^{2} = 9$ $4^{2} = \dots$ $5^{2} = \dots$ $6^{2} = 36$ [1] (a) Work out (i) 9^{2} , (ii) 40^{2} . [1]

(b) Show that $9^2 + 40^2 = 41^2$.

[2]

3

2

When $a^2 + b^2 = c^2$ then (a, b, c) is a 3-square set. a, b and c are positive integers.

Example In Question 2(b), a = 9, b = 40 and c = 41. $9^2 + 40^2 = 41^2$, so (9, 40, 41) is a 3-square set. When $a^2 + b^2 = c^2$ then $c = \sqrt{a^2 + b^2}$.

Use this formula and any patterns you notice to complete the table on the next page for 3-square sets.

b	С
4	5
12	13
24	25
40	41
60	
84	85
112	113
144	
	181
	221
312	313
	4 12 24 40 60 84 112 144

<u>Example</u>	From the table	row two row six	$5^2 + 12^2 = 13^2$ $13^2 + 84^2 = 85^2$

Replace 13^2 in the second equation with $5^2 + 12^2$ from the first equation: $5^2 + 12^2 + 84^2 = 85^2$. So (5, 12, 84, 85) is a 4-square set.

Use the same method with rows from the table to find two more 4-square sets.

3

(....., ,, ,) and (....., ,, ,) [3]

[6]

5 (a) Show that (6, 12, 12, 18) is a 4-square set.

(b) k is any positive integer greater than 1. If (ka, kb, kc, kd) is a 4-square set, then $(ka)^2 + (kb)^2 + (kc)^2 = (kd)^2$.

Show that (a, b, c, d) must also be a 4-square set.

[2]

[2]

- (c) The numbers in the 4-square set (6, 12, 12, 18) have common factors.
 - (i) Find a common factor of 6, 12, 12 and 18 that is greater than 1.

......[1]

(ii) Use (6, 12, 12, 18) and **part (b)** to find a 4-square set where *a*, *b*, *c* and *d* do not have a common factor greater than 1.

(.....) [2]

6 Here is another method for finding a 4-square set (a, b, c, d).

Choose two positive integers *a* and *b* with *a* less than *b*.

Then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ make the 4-square set (a, b, c, d).

- (a) Use this to find a 4-square set when
 - (i) a = 2 and b = 3,

(2, 3,) [3]

(ii) a = 2 and d = 43.

(2,, 43) [3]

(b) (i) Use your answers to **part** (a) and any patterns you notice to complete the table for 4-square sets that start with 2.

a	Ь	С	d
2	3		
2	5	14	15
2	7	26	27
2			43
2			

[3]

(ii) Write down an equation connecting c and d.

(c) When a and b are both even then $c = \frac{a^2 + b^2 - 1}{2}$ and $d = \frac{a^2 + b^2 + 1}{2}$ do not give a 4-square set.

Give an example to show this.

[2]

(d) When *a* and *b* are both odd there are no 4-square sets.

In a 4-square set, d = 23.

(i) Show that $a^2 + b^2 = 45$.

(ii) Find a 4-square set when d = 23.

[1]

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