Mark Scheme (Results)

Summer 2021

Pearson Edexcel International Advanced Subsidiary/Advanced Level
In Pure Mathematics P4 (WMA14/01)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2021

Question Paper Log Number P66008A
Publications Code WMA14_01_2106_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⎣ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1 (a)	$(1+kx)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(kx)+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2+\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3...$	
(i)	$\frac{1}{2}k=\frac{1}{8}\Rightarrow k=\frac{1}{4}$	M1A1
(ii)	$A=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times "k"^2=-\frac{1}{128}$ $\quad B=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times "k"^3=\frac{1}{1024}$	M1 A1 A1
		(5)
(b)	Substitutes $x=0.6\Rightarrow\sqrt{1.15}=1+\frac{1}{8}\times0.6-"\frac{1}{128}"\times0.6^2+"\frac{1}{1024}"\times0.6^3=1.072398$	M1 A1
		(2)
		(7 marks)
1(b) alt	$(1+kx)^{\frac{1}{2}}=1+\left(\frac{1}{2}\right)\times(kx)-\left(\frac{1}{8}\right)\times(kx)^2+\left(\frac{1}{16}\right)\times(kx)^3$ Substitutes $"kx"=0.15$ $\Rightarrow\sqrt{1.15}=1+\frac{1}{2}\times0.15-\frac{1}{8}\times0.15^2+\frac{1}{16}\times0.15^3=1.072398$	M1A1 **(2)**

(a)(i)

M1: Sets $\frac{1}{2}k=\frac{1}{8}$ or $\frac{1}{2}kx=\frac{1}{8}x$ and proceeds to find *k*. Implied by a correct value for *k*

A1: $k=\frac{1}{4}$ oe such as $\frac{2}{8}$ or 0.25

(a)(ii)

M1: Correct attempt at 3rd **or** 4th term. Eg. $Ax^2=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{2!}\times(kx)^2$ or $Bx^3=\frac{\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{3!}\times(kx)^3$ with an attempt at substituting in their value for *k* to find a value for *A* or a value for *B*. Condone a missing bracket around the kx terms so allow with *k* instead of k^2 or k^3 respectively. These may be simplified so award for $A=-\frac{1}{8}\times("k")^2$ or $B=\frac{1}{16}\times("k")^3$ again, condoning *k* instead of k^2 or k^3 respectively.

A1: $A=-\frac{1}{128}$ o.e. There is no requirement to simplify the fraction

A1: $B=\frac{1}{1024}$ o.e You may occasionally see $B=\frac{3}{3072}$ which is fine.

(b)

M1: For an attempt to substitute

- either $kx=0.15$ into an expansion of the form $1\pm p\times(kx)\pm q\times(kx)^2\pm r\times(kx)^3$
- or $x=\frac{0.15}{"k"}$ into their $1+\frac{1}{8}x+"A"x^2+"B"x^3$

A1: 1.072398 Must be to 6 decimal places.

Question Number	Scheme	Marks
2	Achieves a lower limit of 2	B1
	Attempts $\alpha\int y^2\,\mathrm{d}x = \alpha\int \frac{81}{(2x-3)^{2.5}}\mathrm{d}x = \alpha\times\frac{-27}{(2x-3)^{1.5}}$	M1, A1
	Correct attempt at volume of solid generated **under** curve =	
	$=\pi\int_{"2"}^{6} y^2\mathrm{d}x = \left[\frac{k}{(2x-3)^{1.5}}\right]_{"2"}^{6} = \left(\left(-\frac{27}{27}\right)-\left(-\frac{27}{1}\right)\right)$	dM1
	$\text{Volume} = 26\pi$	A1
	Correct attempt at volume of solid $= \pi\times 9^2\times(6-"2")-"26\pi"$	ddM1
	$=298\pi$	A1
		(7 marks)

B1: Finds the lower limit. This may be awarded anywhere. Accept on the diagram or in the limits of an integral

M1: Integrates an expression of the form $\int \frac{...}{(2x-3)^{2.5}}\mathrm{d}x$ and achieves $\frac{...}{(2x-3)^{1.5}}$ oe

A1: Correct integration of $\alpha\int y^2\,\mathrm{d}x = \alpha\int \frac{81}{(2x-3)^{2.5}}\mathrm{d}x$ giving a solution $\alpha\times\frac{-27}{(2x-3)^{1.5}}$ o.e.

following through on their α. Typical values of α will be 1, π or 2π. No need to simplify here

dM1: Attempts to find the volume of the solid generated by rotating the area under the curve.

Look for $\left[\frac{...}{(2x-3)^{1.5}}\right]_{"2"}^{6} = ...$ It is dependent upon the previous M1.

The top limit must be 6 and the bottom limit must be their solution to $\frac{9}{(2x-3)^{1.25}} = 9$

A1: Correct exact volume for the solid generated by rotating the area under the curve. $\text{Volume} = 26\pi$

This may be the second part of a larger/complete expression. E.g. $....-26\pi$

ddM1: Correct method for the volume of the solid formed by rotating the area above the curve.

Accept $\pi\times 9^2\times(6-"2")-"26\pi"$

It is dependent both previous M's

A1: 298π

Question Number	Scheme	Marks
3 (a)	Attempts to find $\dfrac{\frac{1}{3}\times 20^2 \times 24}{160}=20$ seconds	M1 A1
		(2)
(b)	Attempts $\dfrac{\mathrm{d}V}{\mathrm{d}h}=h^2+\dfrac{8}{3}h$	M1
	Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow 160=\left(h^2+\dfrac{8}{3}h\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$	M1 A1
	Substitutes $h=5\Rightarrow 160=\left(5^2+\dfrac{40}{3}\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{96}{23}$ (4.2) cm s^{-1}	dM1 A1
		(5)
		(7 marks)

(a)

M1: Attempts to find the volume, using the given formula, at $h=20$ and dividing by 160. Implied by 20
Condone slips. For example they may have incorrectly multiplied out/calculated the expression for V

A1: 20 seconds. This requires the correct units as well

(b)

M1: For an attempt to differentiate V

Scored for an attempt to multiply out and then differentiate term by term to achieve $\dfrac{\mathrm{d}V}{\mathrm{d}h}=\alpha h^2+\beta h$

or via the product rule to achieve $V=\dfrac{1}{3}h^2(h+4)\Rightarrow\dfrac{\mathrm{d}V}{\mathrm{d}h}=(h+4)\times ph+qh^2$

M1: Attempts to use $\dfrac{\mathrm{d}V}{\mathrm{d}t}=\dfrac{\mathrm{d}V}{\mathrm{d}h}\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$ (or equivalent) with $\dfrac{\mathrm{d}V}{\mathrm{d}t}=160$ and their $\dfrac{\mathrm{d}V}{\mathrm{d}h}$.

A1: Correct expression involving $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ and h E.g. $160=\left(h^2+\dfrac{8}{3}h\right)\times\dfrac{\mathrm{d}h}{\mathrm{d}t}$

dM1: Dependent upon the previous M only. It is for substituting $h=5$ and proceeding to a value for $\dfrac{\mathrm{d}h}{\mathrm{d}t}=...$

A1: Either $\dfrac{96}{23}$ or awrt 4.2 cm s^{-1} There is no requirement for the units here

Question	Scheme	Marks
4.	$\int_1^4 \frac{10}{5x+2x\sqrt{x}}\,dx$	
	$u=\sqrt{x} \Rightarrow x=u^2 \Rightarrow \frac{dx}{du}=2u$	B1
	$\int \frac{10}{5x+2x\sqrt{x}}\,dx = \int \frac{10}{5u^2+2u^3}\,2u\,du$	M1 A1
	$= \int \frac{20}{5u+2u^2}\,du = \int \frac{4}{u} - \frac{8}{5+2u}\,du$	dM1 A1
	$= 4\ln u - 4\ln(5+2u)$	ddM1
	Limits $= \left[4\ln u - 4\ln(5+2u)\right]_1^2 = 4\ln 2 - 4\ln 9 + 4\ln 7 = ...$	M1
	$= 4\ln\left(\frac{14}{9}\right)$	A1 **(8 marks)**

B1: For $\frac{dx}{du}=2u$ or $\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or equivalent

M1: Attempts to write all terms in the integral in terms of u (inc the dx)

Condone slips for this mark BUT the dx CANNOT just be replaced by du.

Look for either $x \to u^2$ or $x\sqrt{x} \to u^3$ with either $dx \to f(u)du$ or $dx \to k \times du,\ k \neq 1$

A1: Correct integrand in terms of just u which may be unsimplified. E.g $\int \frac{10}{5u^2+2u^3}\,2u\,du$

dM1: Attempts to use PF and writes their integrand in terms of its component fractions.

Look for an integral of the form $\int \frac{P}{Qu+Ru^2}\,du \to \int \frac{p}{u} + \frac{q}{Q+Ru}\,du$

A1: Correct PF $\int \frac{4}{u} - \frac{8}{5+2u}(du)$

ddM1: For $...\ln u \pm ...\ln(5+2u)$ but follow through on their PF's which must be of a similar form

M1: Uses the limits 1 and 2 within their attempted integral. The integration may be incorrect.

Alternatively substitutes $u=\sqrt{x}$ and uses the limits 1 and 4 within their attempted integral

A1: $4\ln\left(\frac{14}{9}\right)$

Question Number	Scheme	Marks
5 (a)	$2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$	B1 M1 A1
	$\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3 \Rightarrow \frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y}$ *	A1*
		(4)
(b)	Puts $x=0$ into the equation of the curve $\Rightarrow y = y^2 \Rightarrow y = 1$	B1
	Attempts tangent at $(0,0)$ **or** $(0,1)$ $y = 3x$ **or** $y = -5x+1$	M1 A1
	Solves $y = 3x$ with $y = -5x+1 \Rightarrow R = \left(\frac{1}{8}, \frac{3}{8}\right)$	dM1 A1
		(5)
		(9 marks)

(a) Originally scored M1 B1 A1 A1, now B1 M1 A1 A1

B1: Correct differentiation using the chain rule $y^2 \to 2y\frac{dy}{dx}$.

M1: Attempts to apply the product rule of differentiation on ye^{-2x} to give $e^{-2x}\frac{dy}{dx} \pm ...ye^{-2x}$

A1: Correct differentiation $2y\frac{dy}{dx} = e^{-2x}\frac{dy}{dx} - 2ye^{-2x} - 3$

Allow $2y\,dy = e^{-2x}dy - 2ye^{-2x}dx - 3dx$

A1*: Proceeds to the given answer via an intermediate line equivalent to $\left(e^{-2x} - 2y\right)\frac{dy}{dx} = 2ye^{-2x} + 3$ with correct bracketing.

(b)

B1: Deduces or implies that $x = 0, y = 1$ at P . May state or use coordinates of P (0,1)

M1: Scored for an attempt to find the equation of the tangent at O **or** the equation of the tangent at P

E.g. Substitutes $(0,0)$ into $\frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y} = \beta$ and states $y = \beta x$

Alternatively substitutes $(0,"1")$ into $\frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y} \to "k"$ and attempts $y - 1 = "k"x$ or equivalent

A1: Achieves a correct equation for **either** tangent. Look for **either** $y = 3x$ **or** $y = -5x+1$ o.e.

dM1: Correct attempt **at both** tangents with an attempt to solve simultaneously.

For the attempt to solve accept $y = 3x$, $y = -5x+1 \Rightarrow x = ..., y = ...$

A1: Correct coordinates for $R = \left(\frac{1}{8}, \frac{3}{8}\right)$ oe

Question Number	Scheme	Marks
6 (a)(i)	Area $R = \int y\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t = \int 4\sin t \times -4\sin 2t\,\mathrm{d}t$	M1 A1
	$= -\int 32\sin^2 t\cos t\,\mathrm{d}t$	dM1
	$x = 0 \Rightarrow t = \frac{\pi}{4}$ and $y = 0 \Rightarrow t = 0 \Rightarrow$ Area $= -\int_{\frac{\pi}{4}}^{0} 32\sin^2 t\cos t\,\mathrm{d}t = \int_{0}^{\frac{\pi}{4}} 32\sin^2 t\cos t\,\mathrm{d}t$ *	A1*
		(4)
(ii)	Area $= \left[\frac{32}{3}\sin^3 t\right]_0^{\frac{\pi}{4}} = \frac{32}{3}\times\frac{2\sqrt{2}}{8} = \frac{8\sqrt{2}}{3}$	M1 A1
		(2)
(b)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{x}{2} = 1 - 2\left(\frac{y}{4}\right)^2$	M1A1
	$y = \sqrt{8 - 4x}$	A1
		(3)
(c)	Range is $0 \leqslant \mathrm{f} \leqslant 4$	B1
		(1)
		(10 marks)

(a)(i)

M1: Attempts to multiply y by $\frac{dx}{dt}$ to achieve an integrand of the form $\pm k\sin t\sin 2t$

Look for area $R = \int y\frac{dx}{dt}(dt) = \int (4)\sin t \times \pm k\sin 2t\ (dt)$ but condone a missing 4 or dt

A1: Correct integrand. Achieves area $R = \int y\frac{dx}{dt}(dt) = \int 4\sin t \times -4\sin 2t\,(dt)$

Condone a missing dt and do not be concerned by the limits. Allow unsimplified.

dM1: Substitutes $\sin 2t = 2\sin t\cos t \rightarrow$ seen or implied $= \pm\int A\sin^2 t\cos t\ dt$

Dependent upon previous M. Condone a missing dt and do not be concerned by the limits.

A1*: Achieves area $\left(= \int_0^2 y\ dx\right) = -\int_{\frac{\pi}{4}}^{0} 32\sin^2 t\cos t\ dt \rightarrow \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t\ dt$

This is a given answer so you must see

- evidence of the dt in at least one line other than the given answer
- correct application of the limits as seen above with as a minimum $-\int_{\frac{\pi}{4}}^{0} ...\ dt \rightarrow \int_0^{\frac{\pi}{4}} ...\ dt$

(a)(ii)

M1: Integrates to a form $\left[A\sin^3 t\right]$ with some attempt to apply the limit(s)

You may see a substitution $u = \sin t$ which is fine. Just look for Au^3 with some attempt to apply the adapted limits

A1: Correct answer $\frac{8\sqrt{2}}{3}$ o.e. following correct **algebraic** integration.

Cannot be scored without the M mark.

(b)

M1: Attempts to use a double angle formula of the form $\cos 2t = \pm 1 \pm 2\sin^2 t$ with the parametric equations to get a Cartesian equation.

If the parametric equations are substituted into the given form of the answer $y = \sqrt{ax+b}$, marks are only scored when the double angle formula is used

A1: Any correct un-simplified equation $\frac{x}{2} = 1 - 2\left(\frac{y}{4}\right)^2$

On the alternative method, this A mark is scored when the candidate writes down $a + 2b = 0. - 4b = 16 \Rightarrow a = ..., b = ...$

A1: $y = \sqrt{8-4x}$ or $y = \sqrt{-4x+8}$ ONLY

(c)

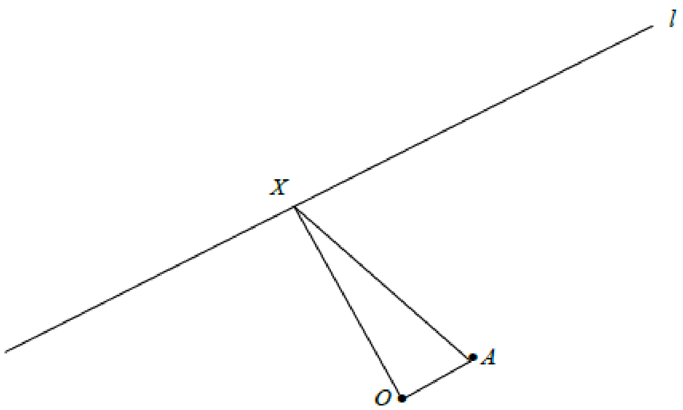
B1: Range is $0 \leqslant \mathrm{f} \leqslant 4$

Allow other acceptable forms such as $0 \leqslant y \leqslant 4$, $0 \leqslant \mathrm{f}(x) \leqslant 4$ and $[0,4]$

Examples of unacceptable forms are $0 \leqslant x \leqslant 4$, $[0,4)$

Question Number	Scheme	Marks
7 (a)	Finds $\left\|\begin{pmatrix}4\\4\\2\end{pmatrix}\right\| = \sqrt{4^2+4^2+2^2} = 6$ and attempts $\frac{1}{"6"}\begin{pmatrix}4\\4\\2\end{pmatrix}$	M1
	$\overrightarrow{OA} = \begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ oe	A1
		(2)
(b)	Co-ordinates or position vector of point $X = \begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$	M1
	$\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix} = 0 \Rightarrow 4(1+4\lambda)+4(-10+4\lambda)+2(-9+2\lambda)=0$	dM1
	$36\lambda = 54 \Rightarrow \lambda = 1.5$	ddM1 A1
	$X = (7,-4,-6)$	A1
		(5)
(c)	Finds $OX = \sqrt{7^2+(-4)^2+(-6)^2} = \sqrt{101}$ and $OA = 1$	M1
	Area $OXA = \frac{1}{2}\times 1\times\sqrt{101} = \frac{\sqrt{101}}{2}$	dM1 A1
		(3)
		(10 marks)

Handy diagram



(a)

M1: Correct attempt at the unit vector. Look for an attempt at $\sqrt{4^2+4^2+2^2}$ and use of $\frac{\mathbf{r}}{|\mathbf{r}|}$ where $\mathbf{r} = \begin{pmatrix}4\\4\\2\end{pmatrix}$ or equivalent vector form such as $\mathbf{r} = (4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$. Condone an attempt to find the coordinates of A

A1: $\overrightarrow{OA}=\begin{pmatrix}2/3\\2/3\\1/3\end{pmatrix}$ or equivalent such as $\overrightarrow{OA}=\frac{1}{6}\begin{pmatrix}4\\4\\2\end{pmatrix}$ or $\overrightarrow{OA}=\frac{1}{6}(4\mathbf{i}+4\mathbf{j}+2\mathbf{k})$ but **must be in vector form and not in coordinate form**.

(b)

M1: For an attempt at the co-ordinates or position vector of point X = $(1+4\lambda,-10+4\lambda,-9+2\lambda)$ **or** $\begin{pmatrix}1+4\lambda\\-10+4\lambda\\-9+2\lambda\end{pmatrix}$

dM1: For using $\overrightarrow{OX}.\begin{pmatrix}4\\4\\2\end{pmatrix}=0$ to set up an equation in λ

Alternatively finds $OX^2=(1+4\lambda)^2+(-10+4\lambda)^2+(-9+2\lambda)^2$ and attempts to differentiate and set = 0

May use $OX^2+OA^2=AX^2$ to set up an equation in λ

ddM1: Solves for λ This is dependent upon having scored both previous M's

A1: Correct value for $\lambda=1.5$

A1: Correct coordinates for $X=(7,-4,-6)$. Condone position vector form $7\mathbf{i}-4\mathbf{j}-6\mathbf{k}$ o.e.

(c)

M1: Finds all elements required to calculate the area.

In the main scheme this would be the distance OX or OX^2 AND distance OA or OA^2 (which is 1)

dM1: Correct method of finding the area of OXA = $\frac{1}{2}\times OX\times 1$

A1: Area = $\frac{\sqrt{101}}{2}$ o.e

....

There are various alternatives for part (c). Amongst others are;

Alt I **via vector product**.

$$\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|=\frac{1}{2}\times\sqrt{\left(\frac{8}{3}\right)^2+\left(-\frac{19}{3}\right)^2+\left(\frac{22}{3}\right)^2}=\frac{\sqrt{101}}{2}$$

M1: For an attempt at $\frac{1}{2}\times\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\7&-4&-6\\2/3&2/3&1/3\end{vmatrix}=\frac{1}{2}\times\left|\frac{8}{3}\mathbf{i}-\frac{19}{3}\mathbf{j}+\frac{22}{3}\mathbf{i}\right|$

dM1: Followed by an attempt at finding the modulus of the resulting vector multiplied by 1/2

Alt II **via scalar products**

M1: Attempts to find all three components required to find the area triangle OXA

E.g. Angle OXA with length of side OX and XA

Alternatively angle OAX with length of side OA and XA

For this to be scored

- appropriate gradient vectors need to be attempted by subtracting
- a correct attempt at using $\mathbf{a.b}=|\mathbf{a}||\mathbf{b}|\cos\theta$ to find $\cos\theta$ or θ

dM1: Full attempt at area of triangle using $\frac{1}{2}|OX||XA|\sin(OXA)$ or equivalent

Question Number	Scheme	Marks
8 (a)	$\int y^{-\frac{1}{3}}\mathrm{d}y = \int 6x\mathrm{e}^{-2x}\mathrm{d}x$	B1
	$\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} + \int 3\mathrm{e}^{-2x}\mathrm{d}x$	M1 M1
	$\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} - \frac{3}{2}\mathrm{e}^{-2x} + c$	dM1 A1
	Substitutes $(0,1) \Rightarrow c = 3$	M1
	$y^2 = \left(-2x\mathrm{e}^{-2x} - \mathrm{e}^{-2x} + 2\right)^3$	A1
		(7)
(b)	As $x \to \infty, \mathrm{e}^{-2x} \to 0$ and $y^2 = (2)^3 \Rightarrow y = 2^{\frac{3}{2}}$	M1 A1
		(2)
		(9 marks)

(a)

B1: Separates the variables either $\int y^{-\frac{1}{3}}\mathrm{d}y = \int 6x\mathrm{e}^{-2x}\mathrm{d}x$ or $\int \frac{1}{6}y^{-\frac{1}{3}}\mathrm{d}y = \int x\mathrm{e}^{-2x}\mathrm{d}x$

Condone with missing integral signs but the dx and dy **must be present** and **in the correct positions**

M1: For integrating the lhs $y^{-\frac{1}{3}} \to y^{\frac{2}{3}}$

M1: For integrating the rhs by parts the right way around. Look for $\int x\mathrm{e}^{-2x}\mathrm{d}x \to ..x\mathrm{e}^{-2x} \pm \int ...\mathrm{e}^{-2x}\mathrm{d}x$

dM1: For fully integrating the rhs to obtain $..x\mathrm{e}^{-2x} \pm ...\mathrm{e}^{-2x}$. Depending upon the previous M

A1: Correct integration with or without '+ c'.

Look for $\frac{3}{2}y^{\frac{2}{3}} = -3x\mathrm{e}^{-2x} - \frac{3}{2}\mathrm{e}^{-2x} + c$ or equivalent such as $3y^{\frac{2}{3}} = -6x\mathrm{e}^{-2x} - 3\mathrm{e}^{-2x} + a$

dM1: Must have a "+c" now. Substitutes $(0,1) \Rightarrow c = ...$

It is dependent upon having a correct attempt to integrate one side so M1 or M2 must have been awarded.

A1: CSO $y^2 = \left(-2x\mathrm{e}^{-2x} - \mathrm{e}^{-2x} + 2\right)^3$

(b)

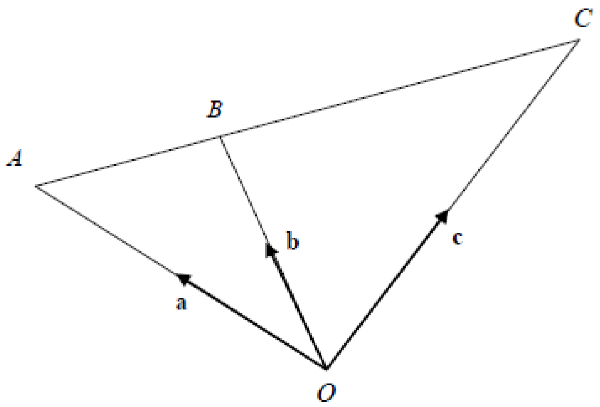
M1: For $\mathrm{e}^{-2x} \to 0$ $y^2 = ("2")^3$

Follow through on their $\mathrm{g}(x)$ in $y^2 = \mathrm{g}(x)$ but $\mathrm{g}(x)$ must be a function of e^{-kx} with $\mathrm{g}(0) \neq 0$

Implied by a correct decimal answer for their $y^2 = \mathrm{g}(x)$

A1: $y = 2^{\frac{3}{2}}$ o.e such as $y = \sqrt{8}$ cso. ISW after sight of this. Condone $y = \pm 2^{\frac{3}{2}}$ o.e.

Question Number	Scheme	Marks
9 (i)	Attempts two of $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ **and** $\overrightarrow{BC} = \mathbf{c} - \mathbf{b}$ either way around Attempts $\mathbf{c} - \mathbf{b} = 2\times(\mathbf{b} - \mathbf{a})$ oe such as $\mathbf{c} - \mathbf{a} = \mathbf{3}\times(\mathbf{b} - \mathbf{a})$ $\Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ *	M1 dM1 A1 * **(3)**
(ii)	Assume that there exists a number n that isn't a multiple of 3 yet n^2 is a multiple of 3 If n is not a multiple of 3 then $m = 3p+1$ or $m = 3p+2$ $(p \in \mathbb{N})$ giving $m^2 = (3p+1)^2 = 9p^2 + 6p + 1$ Or $m^2 = (3p+2)^2 = 9p^2 + 12p + 4 = 3(3p^2 + 4p + 1) + 1$ $(3p+1)^2 = 9p^2 + 6p + 1 (= 3(3p^2 + 2p) + 1)$ is one more than a multiple of 3 $(3p+2)^2 = 9p^2 + 12p + 4$ is not a multiple of 3 as 3 does not divide into 4 (exactly) Hence if n is a multiple of 3 then n^2 is a multiple of 3	B1 M1 M1 A1 A1 **(5)** **(8 marks)**



(i)

M1: Attempts any two of $\overrightarrow{AB}$, $\overrightarrow{AC}$ **and** $\overrightarrow{BC}$**.**

Condone the wrong way around but it must be subtraction.

Allow marked in the correct place on a diagram

dM1: Uses the given information.

Accept $\overrightarrow{AB} = \frac{1}{3}\overrightarrow{AC}$, $\overrightarrow{BC} = 2\times\overrightarrow{AB}$, $\overrightarrow{BC} = \frac{2}{3}\times\overrightarrow{AC}$ etc condoning slips as in previous M1.

A1*: Fully correct work inc bracketing leading to the given answer $\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$

Expect to see the brackets multiplied out. So $\mathbf{c} - \mathbf{b} = 2\times(\mathbf{b} - \mathbf{a}) \Rightarrow \mathbf{c} - \mathbf{b} = 2\mathbf{b} - 2\mathbf{a} \Rightarrow \mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$ is fine.

(ii)

B1: For setting up the contradiction.

Eg Assume that there exists a number n that isn't a multiple of 3, yet n^2 is a multiple of 3

As a minimum accept something like " define a number n such that n is not a multiple of 3 but n^2 is"

M1: States that $m = 3p+1$ or $m = 3p+2$ and attempts to square.

Alternatives exist such as $m = 3p+1$ or $m = 3p-1$

Using modulo 3 arithmetic it would be $1 \to 1$ and $2 \to 4 = 1$

M1: States that $m = 3p+1$ AND $m = 3p+2$ and attempts to square o.e.

A1: Achieves forms that can be argued as to why they are NOT a multiple of 3

E.g. $m^2 = (3p+1)^2 = 3\left(3p^2+2p\right)+1$ or even $9p^2+6p+1$

and $m^2 = (3p+2)^2 = 3\left(3p^2+4p+1\right)+1$ or even $9p^2+12p+4$

A1: Correct proof which requires

- Correct calculations
- Correct reasons. E.g. $9p^2+12p+4$ is not a multiple of 3 as 4 is not a multiple of 3

  There are many ways to argue these. E.g $m^2 = (3p+1)^2 = 3\left(3p^2+2p\right)+1$ is sufficient as long as followed (or preceded by) "not a multiple of 3"
- Minimal conclusion such as ✓. Note that B0 M1 M1 M1 A1 is possible

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom