Pure Mathematics P2 Mark scheme

Question	Scheme	Marks
1(a)	$f(x) = x^4 + x^3 + 2x^2 + ax + b$	
	Attempting f(1) **or** f(−1)	M1
	f(1) = 1 + 1 + 2 + *a* + b = 7 **or** 4 + *a* + b = 7 ⇒ *a* + b = 3 (as required) **AG**	A1* **cso**
		(2)
(b)	Attempting f(−2) **or** f(2)	M1
	f(−2) = 16 – 8 + 8 – 2*a* + *b* = –8 {⇒ – 2*a* + *b* = –24}	A1
	Solving both equations simultaneously to get as far as *a* =… **or** *b* =…	dM1
	Any one of *a* = 9 **or** *b* = −6	A1
	Both *a* = 9 and *b* = −6	A1
		(5)
		(7marks)

Notes:

(a)

M1: For attempting either $f(1)$ or $f(-1)$.

A1: For applying $f(1)$, setting the result equal to 7, and manipulating this correctly to give the result given on the paper as $a + b = 3$. Note that the answer is given in part (a).

Alternative

M1: For long division by $(x - 1)$ to give a remainder in *a* and *b* which is independent of *x*.

A1: Or $\{\text{Remainder} =\}$ $b + a + 4 = 7$ leading to the correct result of $a + b = 3$ (answer given).

(b)

M1: Attempting either $f(-2)$ or $f(2)$.

A1: correct underlined equation in *a* and *b*; e.g. $\underline{16 - 8 + 8 - 2a + b = -8}$ or equivalent, e.g. $-2a + b = -24$.

dM1: An attempt to eliminate one variable from 2 linear simultaneous equations in a and b. Note that this mark is dependent upon the award of the first method mark.

A1: Any one of $a = 9$ or $b = -6$.

A1: Both $a = 9$ and $b = -6$ and a correct solution only.

Alternative

M1: For long division by $(x + 2)$ to give a remainder in *a* and *b* which is independent of *x*.

A1: For $\{\text{Remainder} =\}$ $\underline{b - 2(a - 8) = -8}$ $\{\Rightarrow -2a + b = -24\}$.

Then dM1A1A1 are applied in the same way as before.

Question	Scheme		Marks
2(a)	$S_\infty = \dfrac{20}{1-\frac{7}{8}}$; $= 160$	Use of a correct S_∞ formula	M1
		160	A1
			(2)
(b)	$S_{12} = \dfrac{20\left(1-\left(\frac{7}{8}\right)^{12}\right)}{1-\frac{7}{8}}$; $=127.77324...$ $= 127.8$ (1 dp)	M1: Use of a correct S_n formula with n = 12 (condone missing brackets around $\dfrac{7}{8}$)	M1 A1
		A1: **awrt** 127.8	
			(2)
(c)	$160 - \dfrac{20(1-(\frac{7}{8})^N)}{1-\frac{7}{8}} < 0.5$	Applies S_N (**GP only**) with $a = 20$, $r = \frac{7}{8}$ and "uses" 0.5 and their S_∞ at any point in their working.	M1
	$160\left(\dfrac{7}{8}\right)^N < (0.5)$ or $\left(\dfrac{7}{8}\right)^N < \left(\dfrac{0.5}{160}\right)$	Attempt to isolate $+160\left(\frac{7}{8}\right)^N$ or $\left(\frac{7}{8}\right)^N$	dM1
	$N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{160}\right)$	Uses the law of logarithms to obtain an equation or an inequality of the form $N\log\left(\dfrac{7}{8}\right) < \log\left(\dfrac{0.5}{\text{their } S_\infty}\right)$ or $N > \log_{0.875}\left(\dfrac{0.5}{\text{their } S_\infty}\right)$	M1
	$N > \dfrac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823...$ **cso** $\Rightarrow N = 44$	$N = 44$ (Allow $N \geq 44$ but no $N > 44$	A1 cso
	An incorrect **<u>inequality</u>** statement at any stage in a candidate's working loses the final mark. Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. **BUT** it is possible to gain full marks for using =, as long as no incorrect working seen.		
			(4)
	Alternative: Trial & Improvement Method in (c):		
	Attempts $160 - S_N$ or S_N with at least one value for $N > 40$		M1
	Attempts $160 - S_N$ or S_N with $N = 43$ or $N = 44$		dM1
	For evidence of examining $160 - S_N$ or S_N for **both** $N = 43$ **and** $N = 44$ with **both** values correct to 2 DP Eg: $160 - S_{43} =$ awrt 0.51 and $160 - S_{44} =$ awrt 0.45 or $S_{43} =$ awrt 159.49 and $S_{44} =$ awrt 159.55		M1
	$N = 44$		A1 cso
	Answer of $N = 44$ only with no working scores no marks		
			(4)
			(8 marks)

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>3(a)</td><td><table><tr><td>x</td><td>0</td><td>0.25</td><td>0.5</td><td>0.75</td><td>1</td></tr><tr><td>y</td><td>1</td><td>1.251</td><td>1.494</td><td>1.741</td><td>2</td></tr></table></td><td>B1 B1</td></tr>
<tr><td></td><td></td><td>(2)</td></tr>
<tr><td>(b)</td><td>$\frac{1}{2}\times 0.25$, $\{(1 + 2) + 2(1.251 + 1.494 + 1.741)\}$ o.e.</td><td>B1 M1
A1ft</td></tr>
<tr><td></td><td>$= 1.4965$</td><td>A1</td></tr>
<tr><td></td><td></td><td>(4)</td></tr>
<tr><td>(c)</td><td>Gives any valid reason including
• Decrease the width of the strips
• Use more trapezia
• Increase the number of strips
Do not accept use more decimal places</td><td>B1</td></tr>
<tr><td></td><td></td><td>(1)</td></tr>
<tr><td colspan="3" align="right">(7 marks)</td></tr>
<tr><td colspan="3">Notes:</td></tr>
<tr><td colspan="3">(a)
B1: For 1.494
B1: For 1.741 (1.740 is B0). Wrong accuracy e.g. 1.49, 1.74 is B1B0</td></tr>
<tr><td colspan="3">(b)
B1: Need ½ of 0.25 or 0.125 o.e.
M1: Requires first bracket to contain first plus last values and second bracket to include no additional values from the three in the table. If the only mistake is to omit one value from second bracket this may be regarded as a slip and M mark can be allowed (An extra repeated term forfeits the M mark however) x values: M0 if values used in brackets are x values instead of y values
A1ft: Follows their answers to part (a) and is for {correct expression}
A1: Accept 1.4965, 1.497, or 1.50 only after correct work. (No follow through except one special case below following 1.740 in table).

Separate trapezia may be used: B1 for 0.125, M1 for $\frac{1}{2}h(a+b)$ used 3 or 4 times (and A1ft if it is all correct) e.g. 0.125(1+ 1.251) + 0.125(1.251+1.494) + 0.125(1.741 + 2) is M1 A0 equivalent to missing one term in { } in main scheme.</td></tr>
</table>

Question	Scheme	Marks
4	**A solution based around a table of results**	

n	n^2	n^2+2	
1	**1**	**3**	**Odd**
2	**4**	**6**	**Even**
3	**9**	**11**	**Odd**
4	**16**	**18**	**Even**
5	**25**	**27**	**Odd**
6	**36**	**38**	**Even**

Question	Scheme	Marks
	When n is odd, n^2 is odd (odd $\times$odd = odd) so n^2+2 is also odd	M1
	So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)	A1
	When n is even, n^2 is even **and a multiple** of 4, so n^2+2 cannot be a multiple of 4	M1
	Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"	A1*
		(4)
	Alternative - (algebraic) proof	
	If n is even, $n=2k$, so $\frac{n^2+2}{4}=\frac{(2k)^2+2}{4}=\frac{4k^2+2}{4}=k^2+\frac{1}{2}$	M1
	If n is odd, $n=2k+1$, so $\frac{n^2+2}{4}=\frac{(2k+1)^2+2}{4}==\frac{4k^2+4k+3}{4}=k^2+k+\frac{3}{4}$	M1
	For a partial explanation stating that • either of $k^2+\frac{1}{2}$ or $k^2+k+\frac{3}{4}$ are not a whole numbers. • with some valid reason stating why this means that n^2 +2 is not a multiple of 4.	A1
	Full proof with no errors or omissions. This must include • The conjecture • Correct notation and algebra for both even and odd numbers • A full explanation stating why, for all n, n^2+2 is not divisible by 4	A1*
		(4)
		(4 marks)

Question	Scheme		Marks
5(a)	$(S =)a + (a + d) + \ldots + [a+(n-1)d]$	B1: requires at least 3 terms, must include first and last terms, an adjacent term and dots!	B1
	$(S =)[a+(n-1)d] + \ldots\ldots + a$	M1: for reversing series (dots needed)	M1
	$2S = [2a+(n-1)d] + \ldots\ldots + [2a+(n-1)d]$	dM1: for adding, must have $2S$ and be a genuine attempt. Either line is sufficient. Dependent on 1st M1.	dM1
	$2S = n[2a+(n-1)d]$ $S = \frac{n}{2}[2a+(n-1)d]$ cso	(NB −Allow first 3 marks for use of l for last term but as given for final mark)	A1
			(4)
(b)	$600 = 200 + (N-1)20 \Rightarrow N = \ldots$	Use of 600 with a **correct** formula in an attempt to find N.	M1
	$N = 21$	cso	A1
			(2)
(c)	**Look for an AP first:**		
	$S = \frac{21}{2}(2 \times 200 + 20 \times 20)$ **or** $\frac{21}{2}(200 + 600)$ $S = \frac{20}{2}(2 \times 200 + 19 \times 20)$ **or** $\frac{20}{2}(200 + 580)$ (= 8400 **or** 7800)	M1: Use of correct sum formula with their **integer** $n = N$ or $N - 1$ from part (b) where $3 < N < 52$ and $a = 200$ and $d = 20$. M1: Use of correct sum formula with their **integer** $n = N$ or $N - 1$ from part (b) where $3 < N < 52$ and $a = 200$ and $d = 20$.	M1A1
	Then for the constant terms:		
	$600 \times (52 - "N")$ (= 18600)	M1: $600 \times k$ where k is an integer and $3 < k < 52$ A1: A correct un-simplified follow through expression with their k consistent with n so that $n + k = 52$	M1 A1ft
	So total is 27000	**cao**	A1
	There are no marks in (c) for just finding S_{52}		
			(5)
			(11 marks)

Question	Scheme		Marks
6(i)	$\log_2\left(\dfrac{2x}{5x+4}\right)=-3$ **or** $\log_2\left(\dfrac{5x+4}{2x}\right)=3$ **or** $\log_2\left(\dfrac{5x+4}{x}\right)=4$		M1
	$\left(\dfrac{2x}{5x+4}\right)=2^{-3}$ **or** $\left(\dfrac{5x+4}{2x}\right)=2^3$ **or** $\left(\dfrac{5x+4}{x}\right)=2^4$		M1
	$16x = 5x + 4 \Rightarrow x =$ (depends on Ms and must be this equation or equiv)		dM1
	$x = \dfrac{4}{11}$ or exact recurring decimal $0.\dot{3}\dot{6}$ after correct work		A1 **cso**
	Alternative		
	$\log_2(2x)+3=\log_2(5x+4)$		
	So $\log_2(2x)+\log_2(8)=\log_2(5x+4)$ earns 2nd M1 (3 replaced by $\log_2 8$)		2nd M1
	Then $\log_2(16x)=\log_2(5x+4)$ earns 1st M1 (addition law of logs)		1st M1
	Then final M1 A1 as before		dM1A1
			(4)
(ii)	$\log_a y+\log_a 2^3=5$		M1
	$\log_a 8y=5$	Applies product law of logarithms	dM1
	$y=\dfrac{1}{8}a^5$ **cso**	$y=\dfrac{1}{8}a^5$ **cso**	A1
			(3)
			(7 marks)

Notes:

(i)

M1: Applying the subtraction or addition law of logarithms correctly to make **two** log **terms into one** log term .

M1: For RHS of either 2^{-3}, 2^3, 2^4 or $\log_2\left(\dfrac{1}{8}\right)$, $\log_2 8$ or $\log_2 16$ i.e. using connection between log base 2 and 2 to a power. This may follow an error. **Use of** 3^2 **is M0**

dM1: Obtains **correct** linear equation in x. usually the one in the scheme and attempts $x =$

A1: **cso**. Answer of 4/11 with **no** suspect log work preceding this.

(ii)

M1: Applies power law of logarithms to replace $3\log_a 2$ by $\log_a 2^3$ or $\log_a 8$

dM1: (Should not be following M0) Uses addition law of logs to give $\log_a 2^3 y=5$ or $\log_a 8y=5$

Question	Scheme	Marks
7(a)	Obtain $(x \pm 10)^2$ **and** $(y \pm 8)^2$	M1
	(10, 8)	A1
		(2)
(b)	See $(x \pm 10)^2 + (y \pm 8)^2 = 25\ (= r^2)$ **or** $(r^2 =)\ "100" + "64" - 139$	M1
	$r = 5$*	A1
		(2)
(c)	Substitute $x = 13$ into the equation of circle and solve quadratic to give $y =$	M1
	e.g. $x = 13 \Rightarrow (13 - 10)^2 + (y - 8)^2 = 25 \Rightarrow (y - 8)^2 = 16$ so $y = 4$ **or** 12 **N.B. This can be attempted via a 3, 4, 5 triangle so spotting this and achieving one value for y is M1 A1. Both values scores M1 A1 A1**	A1 A1
		(3)
(d)	$OC = \sqrt{10^2 + 8^2} = \sqrt{164}$	M1
	Length of tangent $= \sqrt{164 - 5^2} = \sqrt{139}$	M1 A1
		(3)
		(10 marks)

Notes:

(a)

M1: Obtains $(x \pm 10)^2$ **and** $(y \pm 8)^2$ May be implied by one correct coordinate

A1: (10, 8) Answer only scores both marks.

Alternative: *Method 2:* From $x^2 + y^2 + 2gx + 2fy + c = 0$ centre is $(\pm g, \pm f)$

M1: Obtains $(\pm 10, \pm 8)$

A1: Centre is $(-g, -f)$, **and so centre is** (10, 8).

(b)

M1: For a correct method leading to $r = \ldots$, or $r^2 =$

Allow $"100" + "64" - 139$ or an attempt at using $(x \pm 10)^2 + (y \pm 8)^2 = r^2$ form to identify $r=$

A1*: $r = 5$ This is a printed answer, so a correct method must be seen.

Alternative:

(b)

M1: Attempts to use $\sqrt{g^2 + f^2 - c}$ or $(r^2 =)"100" + "64" - 139$

A1*: $r = 5$ following a correct method.

(c)

M1: Substitutes $x = 13$ into either form of the circle equation, forms and solves the quadratic equation in y

A1: Either $y = 4$ **or** 12

A1: Both $y = 4$ **and** 12

Question 7 notes *continued*
(d) **M1:** Uses Pythagoras' Theorem to find length OC using their $(10, 8)$ **M1:** Uses Pythagoras' Theorem to find *OX*. Look for $\sqrt{OC^2 - r^2}$ **A1:** $\sqrt{139}$ only

Question	Scheme	Marks
8(a)	Substitutes $x=1$ in C_1: $y = 10x - x^2 - 8 = 10 - 1 - 8 = 1$ and in C_2: $y = x^3 = 1^3 = 1 \Rightarrow (1, 1)$ lies on both curves.	B1
		(1)
(b)	$10x - x^2 - 8 = x^3$ $x^3 + x^2 - 10x + 8 = 0$	B1
	$(x-1)(x^2+2x-8) = 0$	M1 A1
	$(x-1)(x+4)(x-2) = 0 \quad x = 2$	M1 A1
	$(2, 8)$	A1
		(6)
(c)	$\int \left\{ \left(10x - x^2 - 8\right) - x^3 \right\} \mathrm{d}x$	M1
	$= 5x^2 - \frac{x^3}{3} - 8x - \frac{x^4}{4}$	M1 A1
	Using limits 2 and 1: $\left(20 - \frac{8}{3} - 16 - 4\right) - \left(5 - \frac{1}{3} - 8 - \frac{1}{4}\right)$	M1
	$= \frac{11}{12}$	A1
		(5)
		(12 marks)

Notes:

(a)

B1: Substitutes x =nto both $y = 10x - x^2 - 8$ and $y = x^3$ **AND** achieves y =1 in both.

(b)

B1: Sets equations equal to each other and proceeds to $x^3 + x^2 - 10x + 8 = 0$

M1: Divides by $(x - 1)$ to form a quadratic factor. Allow any suitable algebraic method including division or inspection.

A1: Correct quadratic factor $(x^2 + 2x - 8)$

M1: For factorising of their quadratic factor.

A1: Achieves x= 2

A1: Coordinates of B = (2, 8)

(c)

M1: For knowing that the area of R = $\int \left\{ \left(10x - x^2 - 8\right) - x^3 \right\} \mathrm{d}x$

This may also be scored for finding separate areas and subtracting.

M1: For raising the power of x seen in at least three terms.

A1: Correct integration. It may be left un-simplified. That is allow $\frac{10x^2}{2}$ for $5x^2$

Question 8 notes *continued*	
M1:	For using the limits "2" and 1 in their integrated expression. If separate areas have been attempted, "2" and 1 must be used in both integrated expressions.
A1:	For $\frac{11}{12}$ or exact equivalent.